

# External Financing and Customer Capital: A Financial Theory of Markups

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March 2017

## Abstract

We propose a dynamic structural corporate model in which firms face imperfect capital markets and frictional product markets. We highlight the importance of the endogeneity of the marginal value of liquidity in determining the interactions between investment, financing and product price setting decisions. Our primary goal is to develop a financial theory of markups to advance the understanding of two related questions in Macro Finance. One is how financial frictions affect firms' markups, and the other is how nominal frictions impact managers' financial decisions and firms' values. The model implies several testable predictions: (1) financially constrained firms are more inclined to increase their desired markups of products; (2) firms facing larger price stickiness tend to issue less external equity and conduct less big payouts; and (3) a large part of the cost from price stickiness is induced by financial frictions. Lastly, we provide stylized facts consistent with our model's predictions.

Key words: markups, financial frictions, customer markets, price stickiness, cash holdings

JEL codes: E31, E32, G1, G3, L21.

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\*We are grateful for the helpful comments and suggestions from George-Marios Angeletos, Abhijit Banerjee, Ariel Burstein, Hui Chen, Peter Demarzo, Zhiguo He, Leonid Kogan, Chen Lian, Erik Loualiche, Debbie Lucas, Hanno Lustig, Andrey Malenko, David Matsa, Jonathan Parker, Mitchell Petersen, Roberto Rigobon, Alp Simsek, Robert Townsend, Iván Werning, Amir Yaron, as well as the participants in MIT Sloan Finance Lunch Seminar and MIT Economics Macro Lunch Seminar. All errors are our own.

# 1 Introduction

We provide a unified theoretical framework based on financial frictions and imperfect product markets to rationalize the impact of financial slack on markups and offer a set of joint predictions on firms' financing, investment and product price setting behaviors. Our financial theory of markups is motivated by the observation that during the 2007-2009 Great Recession, there was a lack of deflationary pressure on product prices while investment and output suffered large declines in the U.S..<sup>1</sup> This phenomenon is believed to be deeply linked to the controversial debate on the cyclical nature of markup dynamics. Despite the extensive debates on cyclical nature of markups, there are few formal dynamic models that analyze the effect of firms' financial slack on markup dynamics.<sup>2</sup> We construct an analytically tractable dynamic investment model integrating customer markets with financial constraints to explicitly link markups to firms' financial slack.

In the standard corporate theory of investment and external financing/capital structure, the product market is typically assumed to generate exogenous stochastic cash flows, and firms' financial decisions are usually independent of their decisions in the product market (e.g. Fischer, Heinkel and Zechner, 1989; Bolton and Scharfstein, 1990; Leland, 1994; Leland and Toft, 1996; Leland, 1998; Grenadier and Wang, 2005; Manso, 2008; He and Xiong, 2012; DeMarzo et al., 2012; Bolton, Chen and Wang, 2011, 2013; Diamond and He, 2014; He and Milbradt, 2014). A key contribution of our paper to this literature is that our model features endogenous cash flows which are affected by the optimal choice of product prices. As a result, product market decisions are interlinked to investment and financial decisions.

In our model, the manager is knowledgeable about choosing product prices endogenously in an imperfect product market to balance the tradeoff between current profits and future customer base.<sup>3</sup> To set up the frictional environment of product price setting for the manager, on top of the short-term demand effect of product prices (i.e., the intra-temporal demand effect) emphasized in most macroeconomic models, we incorporate the inter-temporal demand effect by introducing the frictional customer market that is originated and formulated in the seminal work of Phelps and Winter (1970). In order to build or keep its customer base, the firm sacrifices its average current profits by strategically reducing its product prices, because a lower product price is more likely to retain existing customers and attract new ones, which could increase its long-term average profits. By contrast, by increasing its product prices, the firm can increase its current profits at the expense of losing customer base, which puts future profits and growth at risk. As emphasized by Rotemberg and Woodford (1993), Gilchrist et al. (2016) and Gourio and Rudanko (2014), the long-term nature of customer base and the upfront-paid

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<sup>1</sup>One recent relevant empirical study is Gilchrist et al. (2016) which exploits firm-level data, constructed by merging the U.S. Producer Price Index (PPI) and Compustat datasets, and finds that firms with weak balance sheets increased their product prices significantly relative to industry averages during the height of the crisis in late 2008.

<sup>2</sup>A few exceptions include Chevalier and Scharfstein (1996) and Gilchrist et al. (2016). However, the existing theories are not rich enough because endogenous investment and cash hoarding are not allowed. In principle, firms could take precautions ahead of time (e.g., by accumulating cash) so as to circumvent financial problems during a crisis.

<sup>3</sup>There are numerous examples of imperfect product markets which arise from customers consumption inertia or imperfect information. Chintagunta, Kyriazidou and Perktold (2001) reveal that several yogurt brands exist large habit effects. Bhattacharya and Vogt (2003) provide both theoretical justification and empirical evidence that the drug industry has consumption inertia. Prices of a new drug are kept low and advertising levels are high early in the life cycle in order to build public knowledge about the drug. As knowledge grows, prices rise and advertising falls. Collado and Browning (2007) present evidence that food outside home, alcohol and tobacco are associated with consumption inertia.

costs for customer acquisition render customer base a form of intangible assets held by the firm. Hence, the manager's product price setting decisions are in part investment decisions.<sup>4</sup> Based on this key intuition, we show that product market frictions have nontrivial implications for the firm's dynamics, and in turn, the firm's price setting behavior is also affected by its financial slack. This links to the the "unified q theory" developed by Bolton, Chen and Wang (2011). While the marginal value of liquidity determines the firm's effective marginal cost of investment, we emphasize that the price setting behavior, as a form of investment in intangible assets, is also determined by a q theory with marginal financing costs incorporated.<sup>5</sup> In this sense, our work extends the "unified q theory" to intangible asset investment.

Our model provides a range of novel testable empirical predictions. The first important result concerns how a firm's financial slack affects its product price setting decisions. When setting its product price, the firm in a customer market always balances the tradeoff between current operating revenue and future growth in customer base. When the firm's financial condition is weak, the marginal value of cash is high, which motivates it to set a higher product price in order to mitigate liquidity problems. On the other hand, with abundant internal funds, the firm will stick to a relatively low price in order to build up its customer base. Our model demonstrates that the time-varying financial slack can generate a strong incentive for firms to manipulate their product prices.<sup>6</sup> This mechanism is reminiscent of the empirical findings in Chevalier and Scharfstein (1996), which documents that during regional and macroeconomic recessions, more financially constrained supermarket chains raise their prices relative to less financially constrained chains. In addition, a recent paper by Gilchrist et al. (2016) uses confidential product price data and finds that during the "Great Recession" in the United States, firms with "weak" balance sheets increased their prices relative to industry averages, while firms with "strong" balance sheets chose lower product prices relative to industry prices.

A second key result concerns the impact of price stickiness on the firm's cash holdings, investment, financial decisions, and the firm's value. Like the price setting behavior in standard New Keynesian models (e.g. Galí, 2008), nominal stickiness prevents the firm from setting its product price to the desired

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<sup>4</sup>The strategy of building up customer base through a lower price is prevalent in the automobile and airline industries. Any airline that recovers from a problem announces discounts to bring back fliers. We have seen Air India and Jet Airways doing it in the past post the pilots' strike. More recently, in February 2015, an India airline company, SpiceJet, announced the launch of a new low fare offer as an attempt to win back the trust of the passengers after recovering from financial distress. In March 2010, Toyota fired the first volley when it announced a variety of discount financing and special lease deals to try to regain market share in the U.S. after a series of recalls. To penetrate in the electric car market, in 2014, Nissan boasts the lowest-price electric car in the U.S., after dropping the base price of a Nissan Leaf by \$6,400 earlier this year.

<sup>5</sup>Thus, our paper is also related to the literature on the relevance of intangible assets, including Hall (2001); Atkeson and Kehoe (2005); McGrattan and Prescott (2010); Eisfeldt and Papanikolaou (2013); Ai, Croce and Li (2013); Belo, Lin and Vitorino (2014); Gourio and Rudanko (2014); Ai and Kiku (2013).

<sup>6</sup>Thus we contribute to the theoretical work that examines markup fluctuations over business cycles. Green and Porter (1984) and Haltiwanger and Harrington (1991), for example, predict that markups are procyclical with respect to demand shocks using game-theoretic models. But countercyclical markups are predicted by many more papers, featuring either implicit collusion (e.g. Rotemberg and Saloner, 1986; Rotemberg and Woodford, 1992; Athey, Bagwell and Sanchirico, 2004; Koszegi and Heidhues, 2008) or customer markets (e.g. Phelps and Winter, 1970; Bils, 1989; Gottfries, 1991; Klemperer, 1995; Chevalier and Scharfstein, 1996; Gilchrist et al., 2016). The empirical evidence on markups so far are still mixed due to the lack of good measures of price-cost margins. For example, Domowitz, Hubbard and Petersen (1986); Machin and Van Reenen (1993); Chirinko and Fazzari (1994); Ghosal (2000); Hall (2012); Braun and Raddatz (2012); Nekarda and Ramey (2013); Gilchrist et al. (2016) find markups to be procyclical. In contrast to these studies, Bils (1987); Murphy, Shleifer and Vishny (1989); Rotemberg and Saloner (1986); Rotemberg and Woodford (1991); Chevalier and Scharfstein (1996); Galí, Gertler and López-Salido (2007); Mazumder (2014) finds markups to be countercyclical.

markup immediately, and the firm resets to the desired markup once it receives a Calvo price resetting opportunity.<sup>7</sup> Our model predicts that the firm facing larger price stickiness is more precautionary in its financial decisions. In particular, it tends to delay the payment of dividends or equity repurchases and issue less equity, resulting in more cash holdings on its balance sheet. Stickier prices increase the marginal value of cash, as the option of raising product prices to boost up current cash revenue becomes more costly. This implies that the firm will have the incentive to hold more cash, delaying the payment of dividends in order to cushion against negative demand shocks. However, although the marginal value of cash is high for the firm facing larger price stickiness, this does not imply that the firm will issue more equity when it is running out of cash. On the contrary, our model predicts that in most cases, the firm has already set a high product price by the time of pursuing external financing. When the product price is stickier, the firm anticipates that it is less likely/more costly to lower its price in the near future when it is out of liquidity problems. Therefore, it issues less equity since the demand for cash (mainly from investment) is low when the product price is high due to a decreasing customer base. We provide empirical evidence that is consistent with these predictions by exploiting 18 industries within the manufacturing sector. We show that the industries that change prices less frequently<sup>8</sup> issue less equity and conduct less repurchases. Moreover, we provide empirical evidence that the firms, whose product markets feature high price stickiness, use disproportionately more funds raised from equity issuance to build up cash reserves rather than make investments.

Our model's predictions about the impact of price stickiness on the firm's value is, at first glance, counter-intuitive. We show that the firm facing a stickier price has a larger value in steady state because it endogenously chooses to hold more cash on its balance sheet. However, this does not imply that price stickiness is good in terms of boosting the firm's value. In fact, holding too much cash is costly in our model. Particularly, our model shows that the firm's enterprise value, which is more relevant for operating efficiency and growth (e.g. [Wernerfelt and Montgomery, 1988](#); [Lang and Litzenger, 1989](#); [Chen and Lee, 1995](#); [Bharadwaj, Bharadwaj and Konsynski, 1999](#)), is lower when its product price is stickier.

Moreover, our theory advances the understanding of the impact of price stickiness on investment. The traditional view is that the firm with a stickier price invests less due to a higher cost of capital (e.g. [Weber, 2014](#)). Our model suggests that the endogeneity of cash holdings is missing in this argument. In our model, the firm facing larger price stickiness indeed faces a higher cost of capital, but because of this, it has a strong incentive to hold more cash on its balance sheet. This, on the one hand, reduces the cost of capital. On the other hand, it boosts the firm's value and increases the return on investment, motivating the firm to invest more. The force of the endogenous cash holdings channel works oppositely to the cost of capital channel, mitigating the impact of price stickiness on the investment rate.<sup>9</sup>

A third new result is that our model implies that the cost of price stickiness is higher when external

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<sup>7</sup>Under the menu cost framework, it is equivalent to the case that once the gain from resetting the product price to the desired markup is equal to the menu cost incurred as a result of price adjustment (see Appendix D).

<sup>8</sup>As shown in [Nakamura and Steinsson \(2008\)](#), product price stickiness is a persistent industry characteristic. If an industry has a high price stickiness index, it means that the firms within that industry, on average, face larger price stickiness.

<sup>9</sup>It should be noted that the impact of price stickiness on the firm's enterprise value is exactly consistent with the interesting empirical findings of [Weber \(2014\)](#) because hoarding cash is costly for the shareholder in our model (i.e., the deadweight cost of holding cash).

financing costs are larger. We call it the “amplification effect” of financial frictions on the cost of price stickiness. In fact, [Ball and Romer \(1990\)](#) study the interaction between real price rigidity based on customer markets and nominal price stickiness. They show that significant nominal rigidities can be explained by a combination of real rigidities and small frictions in nominal adjustment, an amplification mechanism via the real rigidity channel. Our model stresses an additional financial channel. Intuitively, the firm loses more value when it is facing more frictions in external financing because a cash-constrained firm tends to rely more on raising the price to boost cash revenue when facing higher costs of external financing. Consider an extreme case when external financing costs are zero, the firm would always prefer to use external financing to replenish cash and the product price will never be distorted. In this case, the cost of price stickiness is zero in our model since the firm would never have the incentive to reset its price. If external financing costs are infinite, the firm would have to raise its price when running out of cash. Since raising prices is more costly for firms with stickier prices, the decrease in the enterprise value is larger.

The remainder of the paper is organized as follows. [Section 2](#) sets out the model. [Section 3](#) discusses the calibration and quantitative results. [Section 4](#) provides brief empirical evidence in support of the model’s predictions. Finally, [Section 5](#) concludes and provides a brief discussion on the robustness of our main results in a general equilibrium model.

## 2 Model

Our model combines the customer market in the literature of imperfectly competitive product markets (e.g. [Phelps and Winter, 1970](#); [Phelps, 1992](#)), investment in neoclassical growth models (e.g. [Hayashi, 1982](#); [Gomes, Kogan and Zhang, 2003](#)), and the imperfect capital market in structural corporate models (e.g. [Bolton, Chen and Wang, 2011, 2013](#)). In the following, we first describe the firm’s demand dynamics and investment behavior, and then we characterize the evolution of the firm’s customer base following traditional customer market models. Next, we introduce the firm’s external financing costs, cash holding costs and dynamics of cash holdings. Lastly, we formulate the firm’s optimization problem.

**Demand and Investment** Consider a large and mature firm whose gross incremental operating revenue  $dY_t$  during a small interval  $[t, t + dt]$  is

$$dY_t = A_t dQ_t, \tag{2.1}$$

where  $A_t$  represents “effective firm size” and  $dQ_t$  is the nominal incremental demand over the interval  $[t, t + dt]$  per unit of effective size. Effective firm size  $A_t$  can also be interpreted as the firm’s average sales. The nominal incremental demand  $dQ_t$  is exogenous and following a diffusion process with drift

term<sup>10</sup>:

$$dQ_t = (p - \bar{c})\mu\left(\frac{p}{\bar{p}}\right) dt + \sigma dZ_t, \quad (2.2)$$

where  $Z_t$  is a standard Brownian motion under the risk neutral measure. We assume the shock  $dZ_t$  to the incremental nominal demand  $dQ_t$  to be exogenous. The variable  $p$  is the product price charged by the firm, the term  $\bar{c}$  is the marginal cost of production, and  $\bar{p}$  is the industry average price of the product. Note that neither the product price  $p$  nor the marginal cost  $\bar{c}$  loads on the nominal shock. Without loss of generality, we normalize the industry average price to one in our calibration, i.e.,  $\bar{p} \equiv 1$ . The average intratemporal demand (i.e. the demand curve faced by the firm in the short run) is characterized by

$$\mu\left(\frac{p}{\bar{p}}\right) = \mu_A\left(\frac{p}{\bar{p}}\right)^{-\eta}, \quad \text{with } \eta > 1. \quad (2.3)$$

This functional form has been widely adopted in the models with monopolistic pricing such as standard New Keynesian models (e.g. [Phelps and Winter, 1970](#); [Galí, 2008](#)). Basically, it means that setting a higher product price  $p$  relative to the industry average lowers the average demand from existing customers.<sup>11</sup> Moreover, in the spirit of [Phelps and Winter \(1970\)](#), we assume that the firm's nominal profits in the short-run ( $(p - \bar{c})\mu(p)$ ) are increasing, on average, in the product price  $p$ . That is, we require  $p \leq p^* \equiv \frac{\eta}{\eta-1}\bar{c}$ , where  $p^*$  is the optimal static monopolistic product price charged by the firm and  $\frac{\eta}{\eta-1}$  is the static monopolistic markup. This assumption is innocuous, as argued by [Phelps and Winter \(1970\)](#), which states that firms often charge lower markups relative to the static monopolistic one.

We assume that the firm's effective size depends on two major factors including customer base  $m_t$  and capital  $K_t$ , i.e.,

$$A_t = a(m_t, K_t). \quad (2.4)$$

In particular, we assume the functional form of the aggregator  $A(\cdot, \cdot)$  to be Cobb-Douglas:

$$a(m, K) \equiv m^\alpha K^{1-\alpha}, \quad (2.5)$$

where  $\alpha$  is the share of customer base in determining the sales of the firm, conditional on the nominal incremental demand  $dQ_t$ . Our model is an extension of the traditional customer market model (e.g. [Phelps and Winter, 1970](#); [Rotemberg and Woodford, 1993](#)), since we incorporate the firm's capital as a factor influencing sales in addition to the firm's customer base. The Cobb-Douglas aggregator is adopted mainly for tractability.

Capital accumulation follows the standard investment model with quadratic adjustment costs. In particular, we assume

$$dK_t = (I_t - \delta K_t)dt, \quad \text{for } t \geq 0, \quad (2.6)$$

<sup>10</sup>The modeling of demand shocks follows the long history in the literature, such as [Caballero \(1991\)](#). More precisely, the modeling of cash flows  $dQ_t$  is very similar to the models of dynamic investment under dynamic optimal incentive contracting (e.g. [DeMarzo et al., 2012](#)). In those models, managers can control the drift of cash flows by choosing effort level, and here by choosing the product price level. Moreover, those models assume that managers cannot control the volatility of the cash flow process.

<sup>11</sup>This effect can be rationalized using a search model where customers conduct sequential search for the cheapest product and the distribution of search costs is uniform across buyers (e.g. [Carlson and McAfee, 1983](#)).

where  $I_t$  is the gross investment rate on  $[t, t + dt]$  and  $\delta$  is the rate of capital depreciation.

With the gross investment adjustment cost, the firm's incremental net profits after paying the investment cost (denoted by  $dN_t$ ) over the time incremental  $dt$  is given by

$$dN_t = A_t dQ_t - \Gamma(I_t, K_t, A_t) dt, \quad \text{for } t \geq 0, \quad (2.7)$$

where  $\Gamma(I, K, A)$  is the total adjustment cost of investment. We assume that the adjustment cost is homogeneous of degree one in  $I$  and  $K$ , in line with the neoclassical investment literature (e.g. Hayashi, 1982). That is, we assume  $\Gamma(I, K, A) = g(i)A$ , where  $i \equiv I/K$  is the firm's investment capital ratio and  $g(i) = \frac{1}{\zeta\theta}(1 + \theta i)^\zeta - \frac{1}{\zeta\theta}$  is an increasing and convex function. We use the standard investment adjustment cost function in the neoclassical investment literature (e.g. Papanikolaou, 2011). Particularly, we take  $\zeta = 2$ , and the functional form of  $g(i)$  simply becomes

$$g(i) \equiv i + \frac{\theta i^2}{2}, \quad (2.8)$$

where  $\theta$  captures the degree of the adjustment cost.

**The Customer Market and Sticky Price Setting** In addition to the short-term demand effect of product prices (i.e., the intra-temporal demand), we incorporate the inter-temporal demand based on the customer market. The key idea is that reducing the product price not only reduces current profits, as  $p\mu(p)$  is increasing in  $p$ . At the same time, the lower product price is more likely to retain existing customers and attract new ones, hence increasing the firm's future profits. Conversely, by increasing the product price, the firm can raise current profits at the cost of losing customer base, hence jeopardizing its future profits and growth.

Following the literature on the customer market (Phelps and Winter, 1970; Rotemberg and Woodford, 1993), we assume that customers gradually learn about prices elsewhere overtime and probably need to overcome some brand switching costs. Therefore, customers drift toward the cheapest sellers slowly. In particular, we postulate that the evolution of the firm's customer base follows the "customer flow equation":

$$dm_t = h\left(\frac{p_t}{\bar{p}}\right) m_t dt, \quad \text{with } h'(\cdot) < 0 \text{ and } h(1) = 1, \quad (2.9)$$

where  $p_t$  is the product price charged by the firm and  $\bar{p}$  is the industry average price. First, note that the customer flow function  $h$  in (2.9) captures the rate at which customers drift from one firm to others when the firm's product price differs from the industry average. Second, the slow-moving assumption in (2.9) captures information frictions faced by customers when searching for the cheapest price<sup>12</sup> or brand switching costs, in line with the search models of product markets, including Gottfries (1986), Klemperer (1987), Farrell and Shapiro (1988), Beggs and Klemperer (1992) and Farrell and Klemperer (2007), among many others. Third, as in Phelps and Winter (1970), the change in customer base is proportional to existing customer base, which implies that a temporal change in relative product prices can bring a

<sup>12</sup>Phelps and Winter (1970) suggest that customers exchange information about the prices charged by different firms through random encounters.

permanent effect on the firm's customer base. Fourth, combining (2.2), (2.3) and (2.9), we see that the long-run elasticity of demand, which measures the percentage response of the eventual demand to a permanent increase in the product price, is larger than the short-run elasticity of demand  $\eta$ .

For simplicity, we adopt the following functional form to model the customer flow function  $h$ , which is also widely used by other customer market models (e.g. Rotemberg and Woodford, 1993; Choudhary and Orszag, 2007):

$$h\left(\frac{p}{\bar{p}}\right) \equiv \kappa - \kappa\left(\frac{p}{\bar{p}}\right)^\nu \quad \text{with } \kappa > 0, \nu > 0. \quad (2.10)$$

The relative price  $p/\bar{p}$  determines the growth rate of customer base. Given the innocuous normalization  $\bar{p} \equiv 1$ , the marginal change in customer base is  $-\nu\kappa p^{\nu-1}$  when the product price varies. Thus, the quantity  $\nu\kappa$  measures how sensitive customers are to changes in the relative price, which can be interpreted as an inverse measure of information frictions or brand switching costs faced by customers.

Price setting follows the continuous-time version of the staggered price-setting model originally developed by Calvo (1983). We assume that the firm's price resetting opportunities arrive randomly following a Poisson process with intensity  $\zeta$ .<sup>13</sup> Intuitively, within any given period  $[t, t + \Delta t]$ , the firm can reset its product price with probability  $1 - e^{-\zeta\Delta t}$ , independent of the time elapsed since the last adjustment. Thus, the average duration between two consecutive price resetting opportunities is  $\zeta^{-1}$ . Therefore, the intensity parameter  $\zeta$  captures the price change frequency which constitutes a natural index of price stickiness. When price resetting opportunities arrive, the firm is free to reset its product price to either  $p_L$  or  $p_H$ .<sup>14</sup>

**Cash Holdings, External Financing and Liquidation** The firm has access to an imperfect capital market. For simplicity, we assume that the firm uses outside equity as the only source of external funds for investment (e.g. Bolton, Chen and Wang, 2011, 2013). The cost of external financing is captured by a fixed cost and a variable cost which is proportional to the amount of issued equity. We assume that the fixed cost is given by  $\phi A$ , where  $\phi$  is the fixed cost parameter. The fixed financing cost plays a crucial role in generating an option-exercising type of external financing decisions. This not only produces severe nonlinearity in investment and the marginal value of cash, but also strongly incentivizes the firm to increase its product price when the firm is financially constrained. The fixed cost is proportional to effective firm size since this ensures that the firm does not grow out of its fixed cost of issuing equity. Technically, the proportional fixed cost also helps to keep the model homogeneous. In addition to the fixed cost, the firm needs to pay a variable financing cost  $\gamma A$ , for each incremental dollar raised from the

<sup>13</sup>Ball and Romer (1990) jointly model the real price rigidity based on customer markets and nominal price stickiness based on small adjustment costs. They emphasize that substantial nominal rigidity can arise from a combination of real rigidities and small nominal frictions. We would like to point out that the main mechanism of the model that the firm increases its price when being financially constrained does not depend on nominal rigidity. Introducing nominal rigidity enables us to analyze its impact on the firm's value, financing, and investment decisions, etc. We adopt the Calvo rule for technical convenience and to follow the convention in the New Keynesian literature. In Appendix D, we propose an alternative model based on menu costs, which shows that our main results remain valid even if nominal rigidity is modeled in a different way.

<sup>14</sup>We assume that the firm can only choose two different prices for simplicity and clarity. We are able to solve the model with continuous product prices. However, the problem becomes more complicated, as a PDE with free boundary conditions has to be solved. We find that the qualitative results are unchanged. Therefore, enabling the firm to choose among two prices is sufficient to convey the main idea of our theory and also helps clarify the model's mechanism in a coherent way.



capital market.

The firm can also file for bankruptcy, resulting in a liquidation value  $L = \ell A$ , which is proportional to effective firm size. In the event of bankruptcy, shareholders obtain  $L + W$ , where  $W$  is the amount of cash holdings when the firm files for bankruptcy.

The firm optimally chooses the timing and the amount of external equity financing. When the gain from external financing is smaller than the value of liquidation, the firm will file for bankruptcy when running out of cash. Otherwise, the firm will pursue external financing.

Combining the firm's cash inflows from the incremental operating profits net the investment expenditures ( $dN_t$  in (2.7)) with cash inflows from financing policies (given by the cumulative payout  $U_t$  and the cumulative external financing  $H_t$ ), the firm's cash inventory  $W_t$  evolves according to the following equation:

$$dW_t = dN_t + (r - \lambda)W_t dt + dH_t - dU_t, \quad (2.11)$$

where the term  $(r - \lambda)W_t dt$  represents the interest income net of the cash carrying cost; the term  $dH_t$  refers to the cash inflows from external financing; and the term  $dU_t$  refers to the cash outflows to investors.

We define the cash ratio as  $w_t \equiv W_t / A_t$ . We show below that the cash ratio  $w_t$  plays an important role as an endogenous state variable in characterizing the equilibrium. Using Ito's lemma, the law of motion for  $w_t$  within the internal financing region is

$$dw = -w [\alpha h(p) + (1 - \alpha)(i(w, p) - \delta)] dt + \left[ (p - \bar{c})\mu(p) - i(w, p) - \frac{\theta}{2}i(w, p)^2 + (r - \lambda)w \right] dt + \sigma dZ_t. \quad (2.12)$$

It shows that the product price  $p$  affects the cash ratio dynamics through three channels. First, a higher  $p$  has a positive effect on the cash ratio through the "current profit channel", because the term  $(p - \bar{c})\mu(p)$  is increasing in  $p$  on the support  $(0, p^*]$ . Second, a higher  $p$  has a positive effect on the cash ratio through the "long-run growth channel" by changing customer base, because the term  $-w\alpha h(p)$  is increasing in  $p$ . More intuitively, a higher  $p$  leads to a smaller growth rate in the firm's effective size, hence making the cash ratio easier to catch up. Third, the product price  $p$  affects the cash ratio through the "investment channel" as reflected by the term  $-[w(1 - \alpha) + 1]i(w, p) - \frac{\theta}{2}i(w, p)^2$ . In fact, the significance of this channel depends on the impact of the product price on investment  $i(w, p)$ , which is determined both by the current profit channel and the long-run growth channel. As we show in Appendix A, without external financing costs, the firm will always focus on the long-run growth channel and investment will not be affected by the current profit channel. However, when there are external financing costs, the firm concerns more about the current profit channel if its financial slack is not sound.

The firm maximizes shareholders' value, as below, by optimally choosing its investment  $I$ , its product price  $p$ , payout policy  $U$ , external equity financing policy  $H$ , and liquidation time  $\tau$ :

$$\mathbb{E}_0 \left[ \int_0^\tau e^{-rt} (dU_t - dH_t - dX_t) + e^{-r\tau} (\ell K_\tau + W_\tau) \right], \quad (2.13)$$

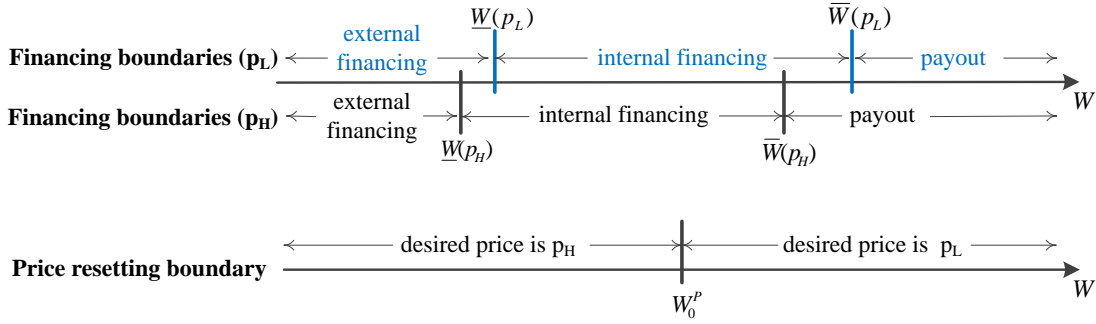


Figure 1: Illustrative Graph for the Decision Boundaries and Regions.

where the expectation is taken under the risk-neutral measure. The term  $dU_t - dH_t - dX_t$  is the discounted value of net payouts to shareholders. The quantity  $X_t$  is the cumulative costs of external financing up to time  $t$ , and  $dX_t$  is the incremental costs of raising incremental external equity  $dH_t$ . The term  $\ell K_\tau + W_\tau$  is the liquidation value paid to shareholders at the time of bankruptcy  $\tau$ .

## 2.1 Model Solution

Let  $U(A, W, p)$  be the value function of the firm. The firm needs to endogenously and simultaneously make three kinds of decisions, namely, investment decisions, financing/liquidation decisions, and price setting decisions. Since both financing/liquidation decisions and price setting decisions are discrete in our model, they can be sufficiently characterized by “decision boundaries”. Figure 1 elaborates this idea: Basically, the firm’s decision-making depends on which of the following four regions the firm finds itself lying in: (1) an external financing/liquidation region within which the firm pursues external financing ( $dH > 0$ ) or liquidation; (2) an internal financing region within which the firm chooses the high product price ( $p_H$ ) once price resetting opportunities arrive; (3) an internal financing region within which the firm chooses the low product price ( $p_L$ ) once price resetting opportunities arrive; and (4) a payout region within which the firm chooses to payout dividends ( $dU > 0$ ). More precisely, it is optimal for the firm to hoard up cash to finance future investment as a result of precautionary motives. When exogenous nominal demand shocks drive cash holdings  $W$  gradually to some low level  $\underline{W}$  (i.e. the “external finance boundary”) such that the current financing costs and the discounted future financing costs are equal, the firm will decide to raise outside equity. The product price setting decision essentially depends on the tradeoff between long-run customer base buildup and short-run profits. When cash holdings  $W$  are lower than  $W_0^P$  (i.e. the “price setting boundary”), the marginal value of cash is large enough so that the marginal value of short-run profits dominates the marginal value of developing future customer base. Thus, the firm desires to raise the product price to increase current profits. Lastly, because holding cash is costly (captured by  $\lambda > 0$ ), the firm chooses to pay out cash when exogenous shocks drive cash holdings  $W$  beyond some high level  $\bar{W}$  (i.e. the “payout boundary”).

**Internal Financing Region** The equilibrium dynamics within the internal financing region can be described by the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned}
rU(A, W, p) = & \max_{I, p^+ \in \{p_L, p_H\}} [\alpha h(p) + (1 - \alpha)(I/K - \delta)] AU_A \\
& + [(r - \lambda)W + A(p - \bar{c})\mu(p) - \Gamma(I, K, A)] U_W + \frac{1}{2}\sigma^2 A^2 U_{WW} \\
& + \zeta [U(A, W, p^+) - U(A, W, p)], \tag{2.14}
\end{aligned}$$

where  $A$  is the effective firm size defined in (2.4). The term  $U_A$  represents the marginal effect of increasing effective firm size on the firm's value, while effective firm size is changing due to net investment ( $I/K - \delta$ ) and the change in customer base ( $h(p)$ ). The term  $U_W$  represents the effect of the firm's expected savings and profits on the firm's value, the term  $U_{WW}$  represents the effect of the volatility of cash holdings on the firm's value, and the jump term represents the jump in the firm's value caused by the change in the product price.

Price resetting decisions amount to comparing the value functions  $U(A, W, p_L)$  and  $U(A, W, p_H)$ . The optimal price is chosen to be  $p_L$  if and only if  $U(A, W, p_L) \geq U(A, W, p_H)$ . The optimal investment rate  $i = I/K$  is pinned down by the following first order condition:

$$1 + \theta i = (1 - \alpha) \frac{U_A(A, W, p)}{U_W(A, W, p)}. \tag{2.15}$$

A key simplification in our setup is that the firm's three-state optimization problem can be reduced to a two-state problem by exploiting homogeneity. We define the function  $u(w, p)$  on  $\mathcal{D} = [0, +\infty) \times \{p_L, p_H\}$  such that

$$U(A, W, p) \equiv Au(w, p), \quad \text{with } w = W/A.$$

Therefore, by taking out the scaling factor  $A$ , the HJB equation in (2.14) can be rewritten into a system of coupled ordinary differential equations:

$$\begin{aligned}
ru(w, p_L) = & (u(w, p_L) - wu_w(w, p_L)) [\alpha h(p_L) + (1 - \alpha)(i(w, p_L) - \delta)] + \zeta \max\{u(w, p_H) - u(w, p_L), 0\} \\
& + u_w(w, p_L) \left[ (p_L - \bar{c})\mu(p_L) - i(w, p_L) - \frac{\theta}{2}i(w, p_L)^2 + (r - \lambda)w \right] + u_{ww}(w, p_L) \frac{\sigma^2}{2},
\end{aligned}$$

and

$$\begin{aligned}
ru(w, p_H) = & (u(w, p_H) - wu_w(w, p_H)) [\alpha h(p_H) + (1 - \alpha)(i(w, p_H) - \delta)] + \zeta \max\{u(w, p_L) - u(w, p_H), 0\} \\
& + u_w(w, p_H) \left[ (p_H - \bar{c})\mu(p_H) - i(w, p_H) - \frac{\theta}{2}i(w, p_H)^2 + (r - \lambda)w \right] + u_{ww}(w, p_H) \frac{\sigma^2}{2},
\end{aligned}$$

where

$$i(w, p) \equiv \left[ \frac{u(w, p)}{u_w(w, p)} - w \right] \frac{1 - \alpha}{\theta} - \frac{1}{\theta}, \quad \text{for } p \in \{p_L, p_H\}. \tag{2.16}$$

**Payout Region** The characterization of the payout boundary is mainly based on the work of Dumas (1991). The firm starts to pay out cash when the marginal value of cash held by the firm is less than the marginal value of cash held by shareholders which is one. Thus, the value matching condition gives the following boundary condition:

$$u_w(\bar{w}(p), p) = 1. \quad (2.17)$$

The payout region is characterized by  $w \geq \bar{w}(p)$  for each  $p$ . Whenever the cash ratio is beyond the boundary, it is optimal for the firm to payout all the extra cash  $w - \bar{w}(p)$  in a lump-sum manner and return its cash holdings back to  $\bar{w}(p)$ . Thus, the firm's value in the payout region has the following form:

$$u(w, p) = u(\bar{w}(p), p) + (w - \bar{w}(p)), \quad \text{when } w \geq \bar{w}(p). \quad (2.18)$$

Lump-sum payouts can occur mainly because payout boundaries are different for different product prices. In our quantitative analysis, we show that  $\bar{w}(p_L) > \bar{w}(p_H)$  under the parameter calibration of interest (see Section 3.1). Moreover, the first-order condition for maximizing the firm's value over constant payout boundaries leads to the smooth pasting or the super contact condition

$$u_{ww}(\bar{w}(p), p) = 0, \quad (2.19)$$

where optimization is achieved at  $\bar{w}(p)$ .

**External Financing/Liquidation Region** Although the firm can raise outside equity any time, it is optimal for the firm to raise equity only when it runs out of cash, which means the external financing boundary  $\underline{w}(p) \equiv 0$ . This is due to the following reasons. First, cash within the firm earns a lower interest rate  $r - \lambda$  due to the cash holding cost. Second, the firm's investment is continuous. Third, financing costs have smaller present value when they are paid further in the future.

Once the firm hits the financing boundary and decides to raise external equity, the optimal financing amount is also endogenously determined. The value matching condition for the issuance amount  $w^*(p)A$  is

$$u(0, p) = u(w^*(p), p) - \phi - (1 + \gamma)w^*(p). \quad (2.20)$$

The left-hand side of equation (2.20) is the firm's value per unit of effective size right before the issuance. The right-hand side of equation (2.20) is the firm's value per unit of effective size minus both the fixed and variable costs of equity issuance per unit of effective size. The first-order optimality condition for the issue amount leads to the smooth pasting condition

$$u_w(w^*(p), p) = 1 + \gamma. \quad (2.21)$$

Since  $w^*(p)$  is the optimal equity issuance, the marginal value of the last dollar raised by the firm must equal to one plus the marginal cost of external financing  $\gamma$ .

Now, we characterize the liquidation boundary  $\underline{w}^L$  and the decision of liquidation. It is easy to see that  $\underline{w}^L \equiv 0$ , since as long as  $w > 0$ , the firm can still invest. In this case, both the marginal value of

effective size and the marginal value of cash within the firm are larger than one, thus the firm's value is strictly larger than the value of liquidation. In the model, when the firm uses up all its cash, it needs to decide whether to issue equity or to file for bankruptcy. If it is optimal for the firm to choose filing bankruptcy instead of raising external equity at the boundary  $\underline{w}^L = \underline{w}(p) = 0$ , the liquidation value per unit of effective size gives

$$u(0, p) = \ell. \quad (2.22)$$

### 3 Quantitative Results

#### 3.1 Parameter Choices and Calibration

We discipline the model by choosing parameter values based on existing calibration and empirical evidence. The liquidation parameter is set to be  $l = 0.9$  following the estimates provided by [Hennessy and Whited \(2007\)](#). We choose the variable cost of financing to be  $\gamma = 6\%$  based on the estimates reported by [Altinkilic and Hansen \(2000\)](#) and the fixed cost of financing is  $\phi = 2\%$ . The interest rate is taken to be  $r_f = 3\%$ , which is within the range of broad empirical evidence in the United States. The volatility of demand shocks is set to be  $\sigma = 12\%$ , which is consistent with the parameters estimated by [Eberly, Rebelo and Vincent \(2009\)](#). The rate of depreciation is set to be  $\delta = 2\%$ . The cash holding cost is assumed to be  $\lambda = 0.9\%$ .<sup>15</sup> The adjustment cost parameter is  $\theta = 1.5$  ([Whited, 1992](#)). We set the Calvo price intensity to  $\xi = 2.8$  to generate a median price duration of 4.3 months as reported in [Bils and Klenow \(2004\)](#). We set  $\eta = 1.5$  as used in [Backus, Kehoe and Kydland \(1994\)](#) and [Zimmermann \(1997\)](#). We set  $\bar{c} = 0.78$ ,  $p_L = 0.95$ , and  $p_H = 2.34$ , implying that  $p_H$  is the price that maximizes cash revenue, namely,  $p_H = \frac{\eta}{\eta - 1} \bar{c}$ , and the industrial average price is about 1.<sup>16</sup> There is no direct empirical evidence on the growth rate of customer base for different prices, we choose  $\kappa = 0.73$  and  $\nu = 1.3$  to reflect that the firm sets its price to  $p_L$  when cash is abundant, in line with the main implication of the customer market literature ([Phelps and Winter, 1970](#); [Phelps, 1992](#)).

In the end, we are left with two parameters, the expected nominal demand,  $\mu_A$ , and the capital share in effective firm size,  $1 - \alpha$ . We calibrate them to match the relevant moments for U.S. public mature large firms during the period of 1998-2012. We interpret effective firm size  $A$  as total sales and capital  $K$  as total assets. According to the Compustat dataset described in [Appendix B](#), the mean cash-sales ratio is 18.45%, and the mean investment-asset ratio is 3.01%. We set  $\mu = 1.02$  and  $\alpha = 0.145$  to match these two moments. [Table 1](#) summarizes the symbols for the key variables of the model and the parameter values in the benchmark case.

<sup>15</sup>We follow [Bolton, Chen and Wang \(2013\)](#) who interpret the cash holding cost as a result of tax disadvantage or agency frictions. Under the simple tax disadvantage interpretation, compared to borrowing the fund, holding cash as retained earnings bears an additional cost 0.9% because the marginal tax rate is about 30% and the interest rate is 3%.

<sup>16</sup>The simulation results indicate that about 3.6% of the time the firm is setting its price to  $p_H$ , implying that the industry average price is equal to 1.

Table 1: Summary of key variables and parameters

Variable	Symbol	Parameters	Symbol	Value
Capital stock	$K$	Risk-free rate	$r$	3%
Cash holding	$W$	Rate of depreciation	$\delta$	2%
Effective firm size	$A$	Mean nominal demand	$\mu_A$	1.02
Customer base	$m$	Volatility of demand shocks	$\sigma$	12%
Investment	$I$	Adjustment cost parameter	$\theta$	1.5
Cumulative nominal demand	$Q$	Share of customer base	$\alpha$	0.145
Cumulative gross operating revenue	$Y$	Marginal cost of production	$\underline{c}$	0.78
Cumulative external financing	$H$	Fixed financing cost	$\phi$	2%
Cumulative payout	$U$	Variable financing cost	$\gamma$	6%
Price resetting boundary	$W_0^P$	Demand elasticity	$\eta$	1.5
External financing boundary ( $p_H$ )	$\bar{W}(p_H)$	Customer base growth parameter	$\kappa$	0.73
External financing boundary ( $p_L$ )	$\bar{W}(p_L)$	Customer base growth parameter	$\nu$	1.3
Payout boundary ( $p_H$ )	$\bar{W}(p_H)$	Proportional cash-carrying cost	$\lambda$	0.9%
Payout boundary ( $p_L$ )	$\bar{W}(p_L)$	Liquidation parameter	$l$	0.9
Optimal financing amount ( $p_H$ )	$W^*(p_H)$	Calvo parameter	$\xi$	2.8
Optimal financing amount ( $p_L$ )	$W^*(p_L)$	High price	$p_H$	2.34
		Low price	$p_L$	0.95

### 3.2 Basic Mechanism: Financial Drivers of Markups

In this section, we elaborate on the rich interactions among cash holdings, product prices, investment, and financing decisions. Note that in our model, there are three channels that the firm can raise short-term cash inflows: increasing product prices, disinvesting, or external financing. However, there are costs associated with each channel either directly incurred or indirectly reflected as a loss in the firm's future revenue. The strategy for a liquidity constrained firm is to choose an optimal combination of the three choices to avoid the possibility of liquidation in the short run, while at the same time taking into account long-run growth opportunities.

**Enterprise Value** The firm's enterprise value is defined as the value of all the firm's marketable claims minus cash,  $U(A, W, P) - W$ , which can be considered as the value of the firm's total tangible and intangible capital stock. We normalize the enterprise value by effective firm size, and obtain  $u(w, p) - w = \frac{U(A, W, P) - W}{A}$ , where  $w = W/A$  denotes the cash-size ratio. This normalized enterprise value  $u(w, p) - w$  can be considered as a measure of the firm's average  $q$ , thus reflecting operating efficiency and growth (e.g. Wernerfelt and Montgomery, 1988; Lang and Litzenberger, 1989; Chen and Lee, 1995; Bharadwaj, Bharadwaj and Konsynski, 1999).

Panel A of Figure 2 plots the normalized enterprise value as a function of the cash-size ratio for the two product prices,  $p_L$  and  $p_H$ , respectively. The solid line represents the normalized enterprise value when the product price is set at  $p_L$ . It is concave and increasing in the region between zero and the payout boundary  $\bar{w}_{p_L} = 0.26$  (the vertical dotted line), and becomes flat (with slope zero) beyond the payout boundary ( $w \geq \bar{w}_{p_L}$ ). The dashed line represents the normalized enterprise value when the product price is set at  $p_H$ , which has a similar shape as the solid line but is associated with a lower payout boundary  $\bar{w}_{p_H} = 0.22$  (the vertical dotted line).

The two curves capturing the normalized enterprise value intersect with each other at the price resetting boundary,  $w_0^P = 0.095$  (the vertical solid line). For  $w > w_0^P$ , the normalized enterprise value is higher if the product price is set at  $p_L$ ; while for  $w < w_0^P$ , the normalized enterprise value is higher for  $p_H$ . This implies that when price resetting opportunities arrive (with Poisson intensity  $\zeta$ ), the firm will set its price to  $p_H$  if the cash-size ratio is less than  $w_0^P$ , and  $p_L$  if the cash-size ratio is larger than  $w_0^P$ . The fact that the optimal product price varies with the cash-size ratio is generated by two forces underlying our model. The product price not only affects short-term operating revenue (the “current profit channel”, captured by  $dQ_t = (p_t - \bar{c})\mu(\frac{p_t}{p})dt$ ), but also determines the growth rate in customer base (the “growth channel”, captured by  $dm_t = h(\frac{p_t}{p})m_tdt$ ). Therefore, there exists a trade-off between  $p_L$  and  $p_H$ . Setting the price to  $p_H$  enables the firm to increase its short-term operating revenue, but customer base will be gradually diminishing. By contrast, the firm builds up its customer base over time by setting the price to  $p_L$ , at the cost of lowering short-term operating revenue. The current profit channel is more crucial when the firm is liquidity constrained (i.e. with a low cash-size ratio), as in this case the marginal value of cash is high. Thus the firm is inclined to set its price to  $p_H$  for  $w < w_0^P$ , relying on the current profit channel to accumulate cash. When cash is abundant, the “growth channel” plays a dominating role in determining the firm’s product price setting, and  $p_L$  would be chosen to build up customer base. This mechanism is reminiscent of the empirical findings in [Chevalier and Scharfstein \(1996\)](#) and [Gilchrist et al. \(2016\)](#) that firms under weak/strong financial conditions tend to increase/decrease product prices relative to industry average prices during a recession.

As we have elaborated before, the firm issues equity only when its cash holdings hit the zero lower bound because of the proportional cash carrying cost and continuous investment flows (see [Bolton, Chen and Wang, 2011](#), for more detailed explanations). At the financing boundaries (i.e.  $\underline{w}_{p_L} = \underline{w}_{p_H} = 0$ ), the firm’s normalized enterprise value is strictly higher than its liquidation value. Therefore, external financing is always preferred to liquidation under our model parameterization.

**Marginal Value of Cash** Panel B plots the marginal value of cash  $u_w(w, p)$  for the two product prices  $p_L$  and  $p_H$ . When the cash-size ratio is beyond the payout boundary, the marginal value of cash is equal to one. The marginal value of cash is higher when the firm becomes more liquidity constrained due to the frictions in external financing. This induces the firm to hoard cash in order to reduce the likelihood of external financing, although holding cash itself is costly, as captured by  $\lambda > 0$ . The frictions in external financing effectively generate “risk aversion” for the firm, a point emphasized by [Bolton, Chen and Wang \(2011\)](#).

To economize on the fixed external financing cost ( $\phi = 2\%$ ), the firm issues equity in lumps. Conditional on issuing equity and having paid the fixed cost, the amount of equity issued returns the cash-size ratio to the point where the marginal value of cash  $u_w(w, p)$  is equal to the marginal cost  $1 + \gamma$ . The firm’s optimal issuance amount is  $w_{p_L}^* = 0.103$  (the vertical dashed line) for  $p_L$ , and  $w_{p_H}^* = 0.066$  (the vertical dashed line) for  $p_H$ . To the left of the optimal issuance amount, the marginal value of cash is higher than  $1 + \gamma$ , reflecting the fact that the fixed external financing cost in raising equity increases the marginal value of cash. If there is no fixed external financing cost ( $\phi = 0$ ), the firm’s optimal issuance amount is zero, as the firm raises just sufficient funds to keep positive  $w$  and to avoid incurring the cash

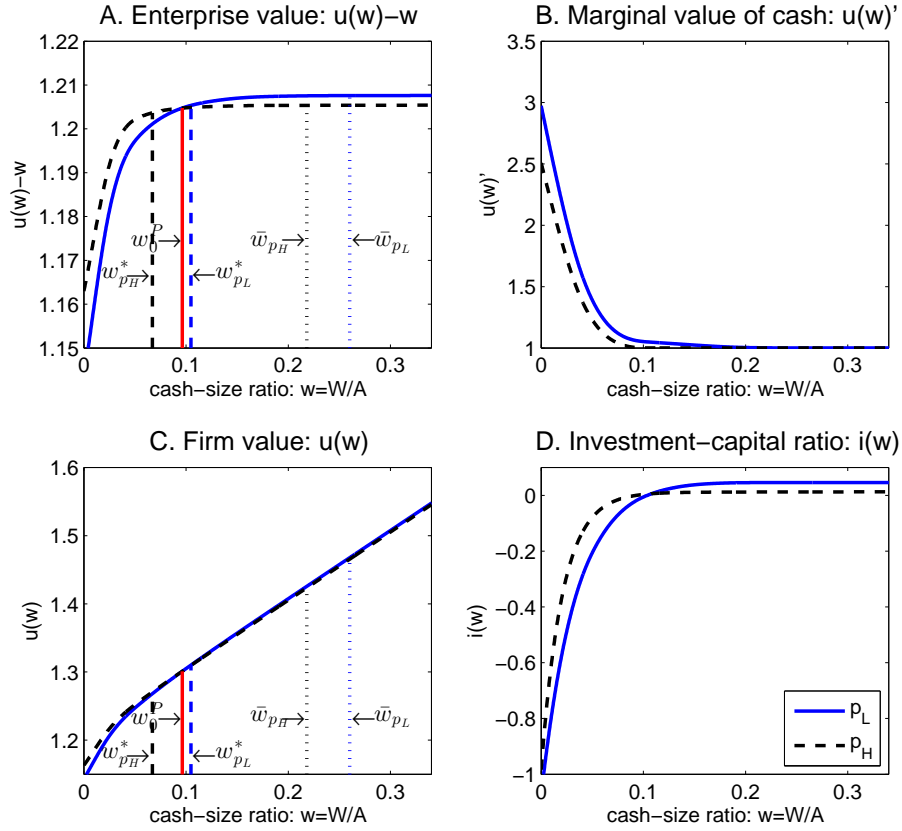


Figure 2: The firm's value, marginal value of cash, and optimal investment rates.

carrying cost.

Note that within the internal financing region ( $\underline{w} < w < \bar{w}$ ), the marginal value of cash is always higher when the product price is  $p_L$ . As a result, the firm with  $p_L$  delays the payout ( $\bar{w}_{p_L} > \bar{w}_{p_H}$ ) and replenishes more cash when issuing equity ( $w_{p_L}^* > w_{p_H}^*$ ) compared to the firm with  $p_H$ . The firm with  $p_L$  faces a higher marginal value of cash because in general it receives less cash inflows. As mentioned above, conditional on the same demand shock, setting a lower product price generates less short-term operating revenue. Since price resetting opportunities arrive in a Poisson fashion, the firm may find itself unable to adjust its product price to  $p_H$  immediately after the cash-size ratio  $w$  drops below the price resetting boundary  $w_0^p$ . This implies that the firm with  $p_L$  is more likely to hit the financing boundary after experiencing a sequence of negative demand shocks. The higher likelihood of executing costly external financing drives up the marginal value of cash. As a result, the firm with  $p_L$  is motivated to delay the payout of equity and endogenously chooses to hold more cash.

Panel C of Figure 2 plots the normalized firm value, which is equal to the enterprise value plus the value of cash.

**Investment** Panel D of Figure 2 plots the firm's optimal investment-capital ratio. The investment-capital ratio is increasing in cash between zero and the payout boundary, and becomes flat beyond the payout



boundary. Notably, the firm disinvests when the cash-size ratio is low. This is to move away from the financing boundary to avoid costly external financing. However, disinvesting is costly not only because of its effect on lowering the growth rate of effective firm size, but also due to the convex capital adjustment cost. Since external financing bears a fixed cost, the firm only issues equity when the cash-size ratio hits the zero lower bound. To avoid paying the fixed cost, the firm starts to raise cash through disinvesting and price adjustment before the cash-size ratio hits zero. Once the firm is completely running out of cash, it raises sufficient cash in lump-sum through the external financing channel. This mechanism delivers a pecking-order solution to liquidity problems: the firm holds cash on its balance sheet to cushion against negative demand shocks. When shocks are small or only last for a few periods, the firm can run down cash and partially rely on raising its price or disinvesting to refill its cash reserves. However, if shocks are large or long lasting, the firm will eventually run out of cash, in which case it would fill up the cash reserve through the external financing channel, which is most costly due to the fixed cost.

Moreover, compared to the firm with  $p_H$ , the firm with  $p_L$  invests more when cash is sufficient; however, it invests less (disinvests more) when cash is constrained. This can be explained by the interaction between the current profit channel and the growth channel. When cash is abundant, the marginal value of cash is one, and the firm purely focuses on the growth channel. The firm with  $p_L$  enjoys a higher growth rate in customer base. Since investment has a complementary effect in boosting the growth rate of effective firm size, the firm with  $p_L$  optimally chooses to invest more.<sup>17</sup> However, if the firm is cash constrained, the current profit channel kicks in and starts to play a more important role in determining the firm's investment-capital ratio. With price stickiness, the firm with  $p_L$  cannot adjust its price to  $p_H$  immediately after its cash-size ratio drops below the price resetting boundary  $w_0^p$ . Therefore, it anticipates less incremental operating revenue in the short term, and an increased likelihood of pursuing costly external financing. Being aware of this anticipation, the firm's investment decision is also more precautionary. In Section 3.3, we analyze the impact of price stickiness and show that a higher degree of price stickiness intensifies the precautionary investment motive, leading to less investment.

**Stationary Distribution** Using the optimal policy rules solved from the model, we simulate the evolution of prices and cash holdings of a single firm for 100 years and plot the stationary distribution of normalized cash-size ratios and investment ratios in Figure 3. Not surprisingly, as shown in panel A, cash holdings are relatively high during most of the time because using the above mentioned three channels (i.e. raising price, disinvesting, and external financing) to raise cash is costly. As a result, the probability mass of the investment ratio,  $i(w)$ , is concentrated around the highest value in the relevant support of  $w$  (see panel B). However, there are periods where the firm is liquidity constrained, and hence making negative investment. The stationary distribution of prices also reveals (not reported here) that the firm sets  $p_L$  during most of the time, to attract customers and build up customer base; while  $p_H$  is set for about 3.6% of the entire simulation period to overcome liquidity problems.

<sup>17</sup>To see the complementarity between investment and customer base in boosting effective firm size, note that the growth rate of effective firm size,  $g_A$ , is determined by both investment,  $i$ , and the growth rate of customer base,  $g_m$ . The Cobb-Douglas form,  $A = m^\alpha K^{1-\alpha}$  indicates that the marginal return of increasing capital stock  $K$  increases with the value of customer base  $m$ .

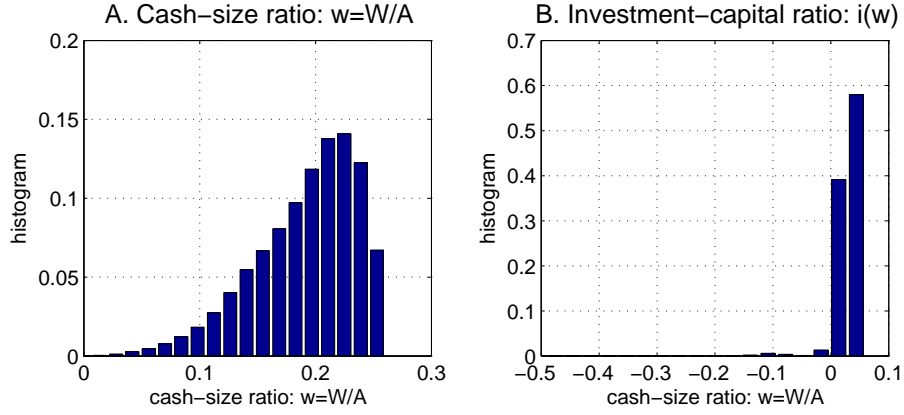


Figure 3: The steady-state distribution of cash and investment.

### 3.3 The Impact of Price Stickiness

In this section, we investigate the impact of price stickiness. Specifically, we analyze the impact of the Calvo parameter  $\zeta$  on the firm's enterprise value, financing, payout, and investment decisions.

**Price Stickiness on the Enterprise Value** Figure 4 compares the firm's enterprise value when the Calvo parameter  $\zeta$  varies. Panel A of Figure 4 plots the benchmark case, with  $\zeta = 2.8$ . The case with a more flexible price ( $\zeta = 40$ ) is shown in Panel B.

The enterprise value for the firm with either  $p_L$  or  $p_H$  is higher in panel B, indicating that price stickiness reduces the enterprise value for any cash-size ratio. This is intuitive since being able to adjust the price can be considered as an option for the firm. The firm always prefers to set its price to  $p_H$  when the cash-size ratio is low ( $w < w_0^P$ ) to take advantage of the current profit channel, and  $p_L$  when the cash-size ratio is high ( $w > w_0^P$ ) to benefit from the growth channel. The likelihood of exercising this option is dependent on the degree of price stickiness, which is captured by the Calvo parameter,  $\zeta$ . The larger  $\zeta$  is, the lower the cost of price adjustment, and the higher the option value and the firm's enterprise value.

Moreover, notice that the enterprise value for the firm with  $p_L$  and the firm with  $p_H$  converges to each other when the price becomes less sticky. This is because when the firm obtains more opportunities to adjust its price, the enterprise value is affected less by the inherited price from the previous instant. On the extreme, when the price is perfectly flexible (with  $\zeta = \infty$ ), the two curves coincide with each other, and the price set in the previous instant no longer matters for the enterprise value (i.e., the product price is no longer a state variable).

**Price Stickiness on Financing and Payout** As shown in Figure 4, for the firm with  $p_L$ , both the payout boundary (the vertical dotted line) and the issuance amount (the vertical dashed line) shift to the left when the price becomes less sticky, because the firm has a larger chance to adjust its product price to  $p_H$  when it is running out of cash. This, as a result, would increase the incremental operating revenue through the current profit channel, and thereby mitigate liquidity problems and decrease the marginal

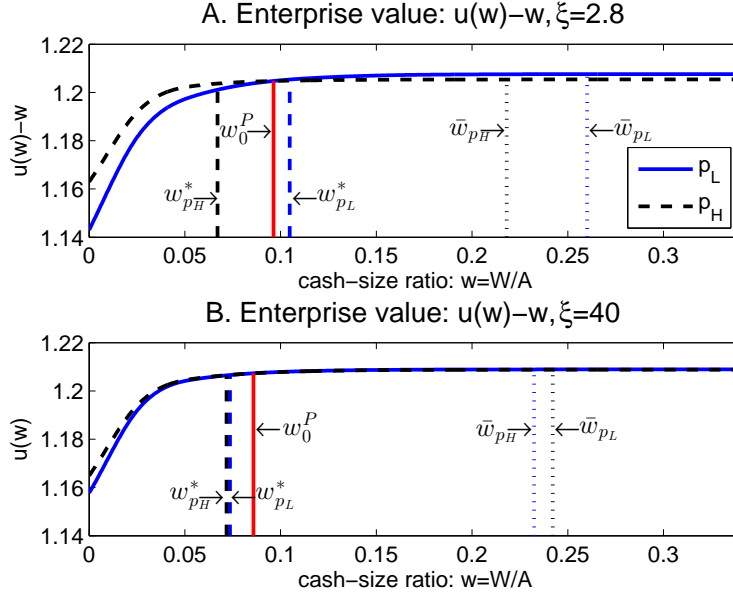


Figure 4: The firm's enterprise values for different value of the Calvo parameter. Panel A is plotted for  $\xi = 2.8$ , and Panel B is plotted for  $\xi = 40$ .

value of cash. Hence, the firm is willing to hold less cash on its balance sheet when facing smaller price stickiness.

On the contrary, for the firm with  $p_H$ , both the payout boundary (the vertical dotted line) and the issuance amount (the vertical dashed line) shift to the right when the price becomes less sticky, indicating that the firm is willing to hold more cash. This is because when the price is less sticky, the firm with  $p_H$  anticipates that, in the future, it is more likely to get the chance to adjust its price to  $p_L$  when its cash-size ratio exceeds the price resetting boundary ( $w_0^P$ ). As mentioned above, setting the low price increases the investment demand for cash due to the complementarity between capital stock and customer base in determining effective firm size. This motivates the firm with  $p_H$  to accumulate more cash, in order to benefit from future investment opportunities through the complementarity channel, when price resetting opportunities arrive.

Price stickiness has diametrically different implications for the firm with  $p_L$  and the firm with  $p_H$  in their financing and payout decisions. This is essentially because the marginal value of cash for the firm with  $p_L$  and the firm with  $p_H$  converges to each other when the price becomes less sticky. As shown in panel B of Figure 2, for any cash-size ratio, the marginal value of cash for the firm with  $p_L$  is higher than that for the firm with  $p_H$ . As a result, the convergence in the marginal value of cash triggered by a smaller price stickiness leads to a decrease in the marginal value of cash for the firm with  $p_L$  and an increase for the firm with  $p_H$ . Since financing and payout decisions are intimately linked to the marginal value of cash, it is not surprising that the firm with  $p_L$  tends to payout more and finance less while the firm with  $p_H$  tends to do the opposite when the price becomes less sticky.

Our simulation results show that for the benchmark calibration, the firm is setting its price to  $p_L$  when repurchasing equity (or issuing dividends) with a probability of 99.5%, thus the model predicts

that firms facing larger price stickiness tend to repurchase equity less frequently, as the payout boundary associated with  $p_L$  in Figure 7 shifts to the right when  $\zeta$  decreases from 40 to 2.8. On the other hand, during 98.5% of time the firm is setting its price to  $p_H$  when pursuing external financing, thus the model predicts that firms facing larger price stickiness issue less equity, as the optimal issuance amount associated with  $p_H$  in Figure 7 shifts to the left when  $\zeta$  decreases from 40 to 2.8.

**Price Stickiness on Investment** Figure 5 presents the firm’s optimal investment decisions when the Calvo parameter varies.

In panel A, the optimal investment ratio is plotted for the firm with  $p_L$ . It is shown that the firm facing smaller price stickiness invests more especially when the cash-size ratio is low. As we noted above, when the cash-size ratio is low, the “current profit channel” constrains the firm with  $p_L$  from investing more. Smaller price stickiness dampens the impact of this channel, as it becomes more likely for the firm to adjust its product price to  $p_H$ , hence boosting short-term operating revenue. Therefore, the liquidity-constrained firm with  $p_L$  increases its investment (or disinvests less) when the price becomes less sticky. For a cash-abundant firm, investment is determined mostly by the “growth channel”, which is not affected much by the degree of price stickiness. Therefore, there is no significant difference in investment among the cash-abundant firms with  $p_L$  when price stickiness varies.<sup>18</sup>

In panel B, we plot the optimal investment ratio for the firm with  $p_H$ . Again, it is shown that the firm facing smaller price stickiness invests more for any cash-size ratio. This is mainly due to the increase in the normalized enterprise value for the firm with  $p_H$  when the price becomes less sticky (see Figure 4). To see this, note that the firm’s value is equal to the sum of the enterprise value and cash, which is equal to the sum of the normalized enterprise value and the cash-size ratio multiplied by the firm’s effective size. Since investment increases effective firm size, its return is higher when either the normalized enterprise value or the cash-size ratio is larger. Therefore, a higher normalized enterprise value due to smaller price stickiness motivates the firm to make more investments.

To illustrate the impact of price stickiness on investment more clearly, in Figure 6, we plot both the optimal investment ratio and the steady-state distribution of the cash-size ratio. For expositional purposes, we also mark the simulated average investment ratio on the figure, and place the up-arrow at the position representing the average cash-size ratio. It shows that the average investment ratio for the baseline calibration ( $\zeta = 2.8$ ) is 0.030, whereas it increases to 0.034 if the firm faces a more flexible price ( $\zeta = 40$ ). These calculations are subject to numerical errors, but the observed small difference indicates that the impact of price stickiness on investment is not large.

There are three forces underlying our model which affect investment in different directions, and as a result, investment is on average not affected significantly by the degree of price stickiness faced by the firm. As we have said above, investment boosts effective firm size, and thus its marginal return is linked to the firm’s value per unit of effective size, which is equal to the sum of the normalized enterprise value and the cash size ratio. The first force reduces investment as the enterprise value is lower when the price is stickier, which decreases the marginal return on investment (see Figure 5). However, there is a

<sup>18</sup>Note that for the firm with  $p_L$ , when cash is abundant, investment ratios are marginally higher when the price is less sticky. This is due to the increase in the enterprise value generated by smaller price stickiness (Figure 4).

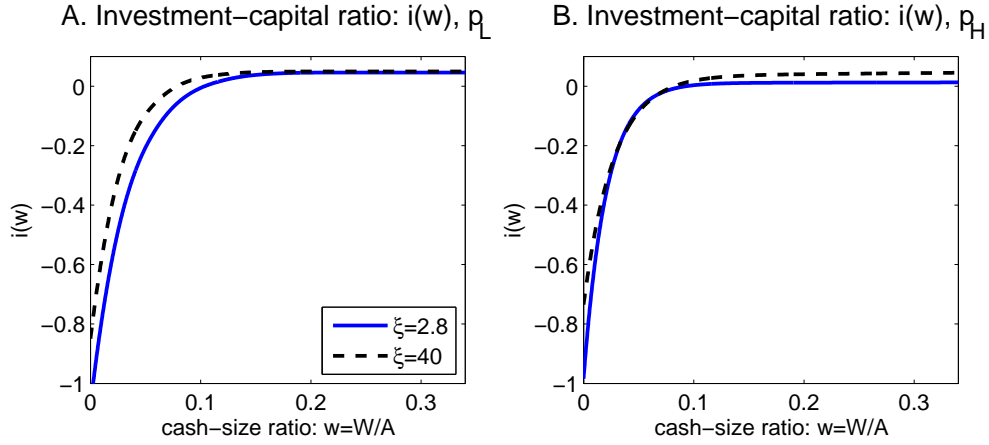


Figure 5: The firm’s investment for different values of the Calvo parameter. Panel A/B plots investment for the  $p_L/p_H$  case. Solid/Dashed lines refer to the  $\xi = 2.8/\xi = 40$  case.

countervailing force transmitted through endogenous cash holdings which pushes a stickier firm to make more investments. In fact, a stickier firm is more precautionary and holds more cash during most of the time, which increases its average cash-size ratio. Since the marginal return on investment also increases with the cash-size ratio, the firm with a stickier price tends to invest more due to more cash holdings. This can be directly seen from Figure 6, although investment for the firm with a stickier price ( $\xi = 2.8$ ) is lower for any cash-size ratio, the distribution of cash holdings is more right skewed. Third, as shown in Figure 5, the difference in investment ratios when price stickiness varies is quantitatively large only for the firm with  $p_L$  when the cash-size ratio is low and for the firm with  $p_H$  when the cash-size ratio is high. However, during most of the time the firm with a low cash-size ratio is setting its price to  $p_H$  and the firm with a high cash-size ratio is setting  $p_L$ . Therefore, the difference in the average investment ratio may not be significant. The latter two results are related to the steady-state distribution of the firm’s cash holdings, which has been largely ignored in the existing literature. Our result complements the traditional view that firms facing larger price stickiness invest less due to the higher cost of capital (e.g. Weber, 2014). In fact, the negative impact of price stickiness on investment could be largely mitigated due to the channel of endogenous cash holdings.

**Quantifying the Cost of Price Stickiness** To shed light on the impact of price stickiness on the firm’s value, we simulate the model for 100 years and compute the average normalized firm value, cash-size ratio, and normalized enterprise value, respectively, over the whole simulation period. We focus on the steady-state outcomes and discard the simulated path of the first 10 years as burn in. Panel A of Figure 7 shows that the firm’s value increases with the degree of price stickiness (decreases with the Calvo parameter). The firm facing larger price stickiness bears a higher marginal value of cash, and is more precautionary in its payout decisions. As a result, it endogenously chooses to hold more cash on its balance sheet, boosting up the firm’s value.<sup>19</sup> This is confirmed in panel B of Figure 7, which shows that cash holdings increase when the price becomes stickier. However, note that the firm facing

<sup>19</sup>Note that the results presented in Panel A of Figure 7 are about the average firm’s value when the firm is in steady state. A stickier price indeed reduces the firm’s value at the impact of the change in price stickiness.

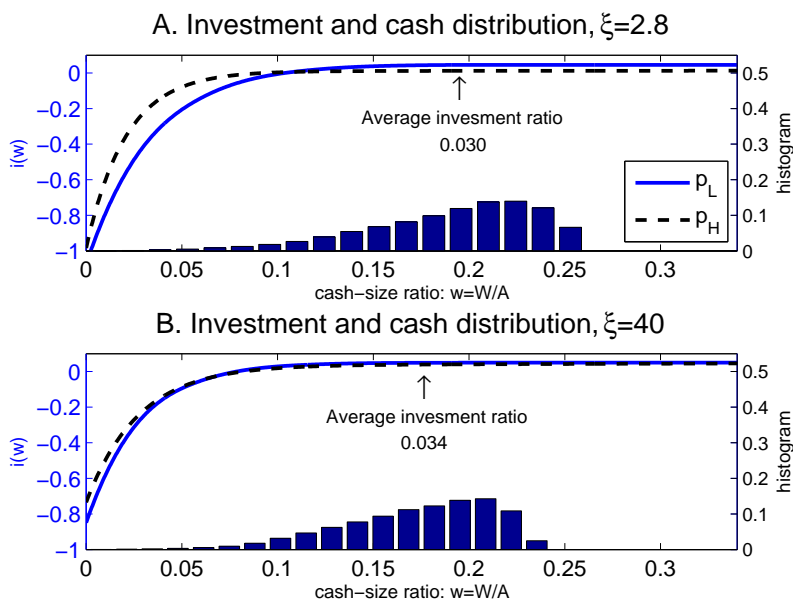


Figure 6: The firm’s investment and cash-size ratio distribution for different values of the Calvo parameter. Panel A is plotted for  $\zeta = 2.8$ , and Panel B is plotted for  $\zeta = 40$ . The left y-axis is for the firm’s investment and the right y-axis is for the steady-state distribution of the cash-size ratio. The up-arrow in each panel is positioned at the average cash-size ratio.

larger price stickiness has a lower enterprise value (panel C of Figure 7). Since the enterprise value is equivalent to the average Tobin’s  $q$  in our model, this implies that the firm with a stickier price is less efficiently operated (e.g. Wernerfelt and Montgomery, 1988; Lang and Litzenger, 1989; Chen and Lee, 1995; Bharadwaj, Bharadwaj and Konsynski, 1999) or riskier (e.g. Liew and Vassalou, 2000; Griffin and Lemmon, 2002). This result is consistent with Weber (2014), which finds that firms facing larger price stickiness are riskier and demand a higher risk premium.

Quantitatively, our simulation results shown in Figure 7 indicate that the firm facing a completely sticky price ( $\zeta = 0$ ) on average holds 14% more cash than the firm facing a perfectly flexible price ( $\zeta = \infty$ ), while the enterprise value is lowered by 0.5%. A sticky price is costly not only through its direct effect on reducing the enterprise value, but also by indirectly inducing the firm to carry more cash, which is costly as captured by  $\lambda > 0$ . Moreover, note that although the reduction in the enterprise value due to price stickiness has a smaller magnitude on average, it is magnified especially when the firm is liquidity constrained. As shown in Figure 4, when the cash-size ratio is near zero, the enterprise value of the firm with  $p_L$  is decreased by about 1.5% when the value of the Calvo parameter decreases from  $\zeta = 40$  to  $\zeta = 2.8$ .

### 3.4 The Interaction Between Price Stickiness and Financial Frictions

In this section, we seek to understand the interaction between price stickiness and financing costs and their joint impact on the firm’s value.<sup>20</sup>

<sup>20</sup>The impact of financing costs is similar to that of Bolton, Chen and Wang (2011) (see Appendix C.1).

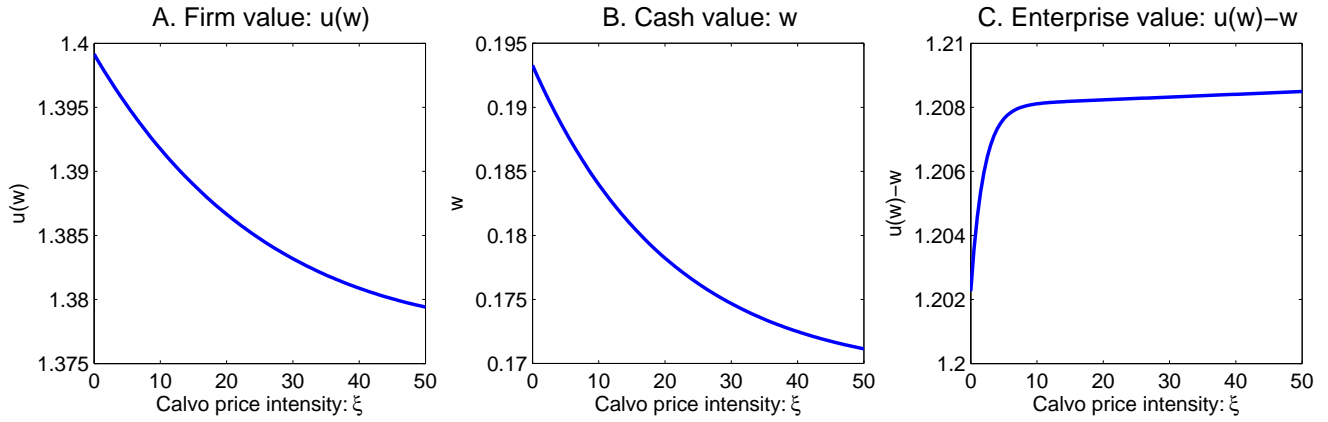


Figure 7: The average steady-state normalized firm value, cash holdings and normalized enterprise value for different values of the Calvo parameter.

Panel A of Figure 8 presents the firm's normalized enterprise value when the fixed financing cost varies from  $\phi = 0$  to  $\phi = 3\%$  for the firm with  $\bar{\zeta} = 2.8$  and  $\bar{\zeta} = 40$ , respectively. Confirming our previous results, the enterprise value is lower as the price becomes stickier. This is true regardless of the value of the fixed financing cost. As shown in panel A, the firm with  $\bar{\zeta} = 2.8$  has uniformly a lower enterprise value (solid line) than the firm with  $\bar{\zeta} = 40$  (dashed line).

Notably, the decrease in the enterprise value due to larger price stickiness increases with the value of the fixed financing cost. This implies that the firm loses more value when it is facing more frictions in external financing. This is because cash-constrained firms tend to rely more on raising prices to boost cash revenue when facing higher costs of external financing. Consider an extreme case when external financing costs are zero, the firm will always prefer to use external financing to replenish cash and stick to the optimal price forever. As shown in panel A, the two curves converge to each other as the fixed financing cost approaches zero, implying that the degree of price stickiness has no impact on the enterprise value. If external financing costs are infinite, the firm will have to raise its price when running out of cash since external financing is not feasible. The firm facing a stickier price is more likely to hit the liquidation boundary, because it is less likely to find a chance to reset its price when the cash-size ratio is low. While the firm facing a more flexible price can timely increase its price and boost cash revenue to avoid liquidation. Hence, the difference in the enterprise value between the two firms, or the cost of price stickiness, is particularly large when external financing is not allowed.

In our model, the cost of holding cash is captured by parameter  $\lambda > 0$ . Panel B of Figure 8 presents the increase in the cash carrying cost when the Calvo parameter decreases from  $\bar{\zeta} = 40$  to  $\bar{\zeta} = 2.8$ . When external financing costs are zero, the firm's cash management policy is no longer affected by price stickiness, thus there is no change in the cash carrying cost when the price becomes stickier. However, for a relatively large fixed external financing cost,  $\phi = 3\%$ , the cost of holding cash increases by about 10% when  $\bar{\zeta}$  is reduced from 40 to 2.8.

In sum, the impact of price stickiness varies significantly with financing costs. Price stickiness becomes more costly for the firm when the frictions in external financing are more severe.

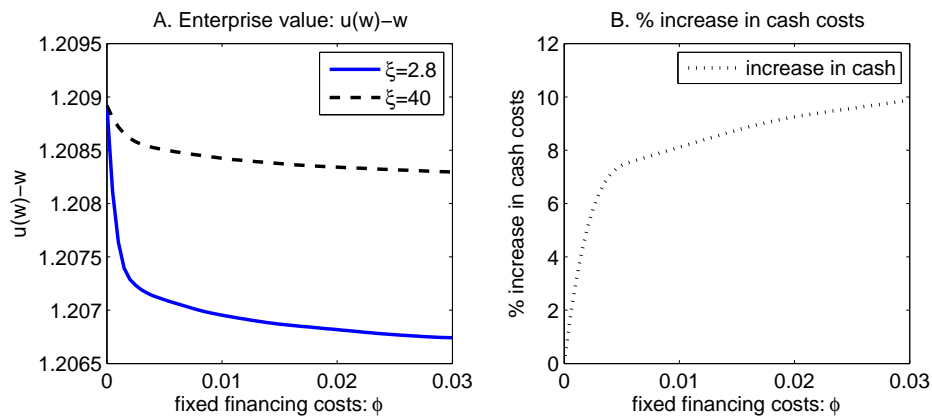


Figure 8: The interaction effect between price stickiness and financing costs on the enterprise value.

## 4 Empirical Evidence

The model makes a number of testable predictions on the interaction between price stickiness and financial frictions, and how they jointly affect the firm’s financial and investment decisions.

The first set of predictions is about how financial frictions and firms’ financial slack affect their product price setting behavior. Our model predicts that financially constrained firms tend to raise their prices in order to increase cash flows. Moreover, the incentive to manipulate prices is larger when prices are less sticky or external financing costs are high. This prediction is supported by the empirical evidence from [Chevalier and Scharfstein \(1996\)](#), which find that during regional and macroeconomic recessions, more financially constrained supermarket chains raise their prices relative to less financially constrained chains. In addition, a recent paper by [Gilchrist et al. \(2016\)](#) uses confidential product-level price data and finds that during the “Great Recession” in the United States, firms with “weak” balance sheets increased their prices relative to industry averages, while firms with “strong” balance sheets lowered prices.

The second set of predictions is about how price stickiness affects firms’ cash holdings and financial decisions. Our model predicts that firms facing larger price stickiness are more precautionary in their financial decisions. In particular, they tend to delay the payment of dividends or equity repurchases (see [Figure 4](#) for the shift of the payout boundary associated with  $p_L$ ) and issue less equity (see [Figure 4](#) for the shift of the optimal issuance amount associated with  $p_H$ ). Below, we provide the empirical evidence that are consistent with these predictions. Due to the lack of firm-level price data, our analysis focuses on the 18 industries within the manufacturing sector. Firms in the same industry produce similar products, and the price stickiness facing them is a persistent industry characteristic, as shown in [Nakamura and Steinsson \(2008\)](#). Hence, our analysis at the industry level is also informative about large firms’ behavior.

The industries are defined to be consistent with the categories used by the Bureau of Economic Analysis (BEA), which include 10 industries producing durable goods and 8 non-durable goods industries. The measure of price stickiness is obtained following the empirical literature, which proxies the degree of stickiness by measuring how frequently firms change their prices (e.g. [Cecchetti, 1986](#); [Baharad and Eden, 2004](#); [Bils and Klenow, 2004](#); [Nakamura and Steinsson, 2008](#)). Firms facing larger price stickiness adjust their prices less frequently. In this sense, the average frequency of price change can be regarded



as an inverse measure of price stickiness. Among the industries we consider, 14 industries are broadly consistent with the major groups defined in Nakamura and Steinsson (2008). Therefore, we use the median frequency of industry-level price change for these 14 industries from Table VI of Nakamura and Steinsson (2008), and the frequency of price change for the rest 4 industries are estimated based on the regressions elaborated in Appendix B.3. The industry-level financing, investment, and other variables used in our analysis are constructed from the Compustat and CRSP quarterly dataset over the period 1998 – 2012. Our analysis starts with year 1998, to be consistent with the price stickiness measure obtained from Nakamura and Steinsson (2008) and the industry categories defined by BEA.<sup>21</sup> We focus on large and mature firms only and thereby exclude the bottom 30% firms sorted by asset value in each industry. All the details are provided in Appendix B.2.

We construct industry-level average equity repurchase ratios and equity issuance ratios from the Compustat and CRSP datasets. For each firm, the equity repurchase ratio in period  $t$  is measured as the increase in the firm’s treasury stock divided by the value of total assets. The equity issuance ratio is measured as the number of common shares issued multiplied by the stock price at the end of the quarter over the value of total assets. The industry-level ratios of equity repurchases and issuances are computed as the average firm-level repurchase and issuance ratios weighted by sales (see Appendix B.2 for more details).

The empirical specifications being employed are (4.1) and (4.2):

$$REPR_i = \beta_0^{rep} + \beta_1^{rep} \log(FREQ_i) + \beta_2^{rep} X_i + \epsilon_i^{rep}, \quad (4.1)$$

$$\Delta EIR_i = \beta_0^{ei} + \beta_1^{ei} \log(FREQ_i) + \beta_2^{ei} X_i + \epsilon_i^{ei}. \quad (4.2)$$

We regress industry-level average equity repurchase ( $REPR_i$ ) and issuance ratio ( $\Delta EIR_i$ ) on the log industry-level price change frequency ( $FREQ_i$ ) controlling for a set of control variables ( $X_i$ ), including the change in average debt ratios ( $\Delta DEBTR_i$ ), average industry stock return ( $R_i$ ), and average abnormal return ( $AR_i$ ).

Figure 9 shows that industries that change prices more frequently (thus face smaller price stickiness) issue more equity and conduct more repurchases during the sample period, consistent with the model’s predictions. The results are significant at the 5% level.

Next, we zoom in the consequence of the linkage between equity financing, repurchases, and price stickiness. Based on a simple regression analysis, we show that for firms facing stickier prices, a larger chunk of the free cash flows obtained from equity financing is held on their balance sheets rather than being invested. This is consistent with our model’s predictions, as firms facing stickier prices have the tendency to build up cash reserves when markets are favorable to cushion against the deterioration of their balance sheet conditions during recession periods. The motive of “save for a rainy day” is inherently generated by the firms with stickier prices, because they are more restricted from using the “raising price channel” to boost up short-term operating revenue.

<sup>21</sup>BEA adopted new industry categories after 1998.

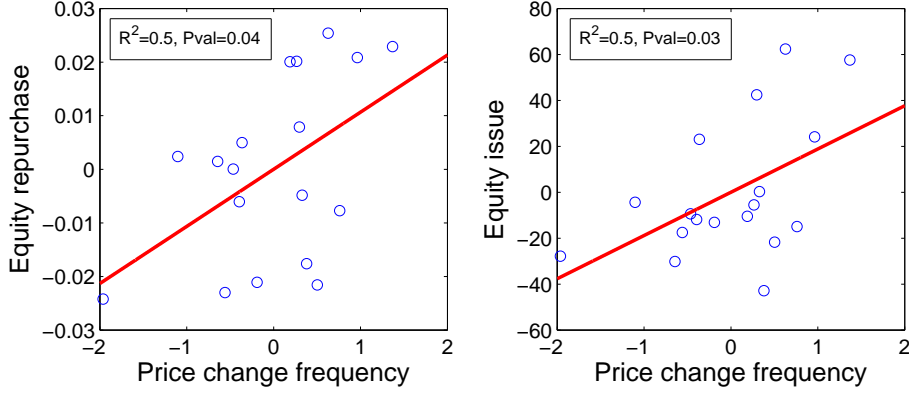


Figure 9: Equity financing, repurchases, and price stickiness. Each dot in the figure characterizes one industry. The solid line is the linear curve fitting of all the dots. The y-value of the dots in the left/right panel are obtained as the residuals from regressing the industry-level average equity repurchase/issuance ratio on the change in average debt ratios, the average industry stock return, and the average abnormal return. The x-value of the dots are obtained as the residuals from regressing the log industry-level price change frequency on the same set of controls. The slope of the fitted lines in the left/right panel is equivalent to the value of coefficient  $\beta_1^{rep} / \beta_1^{ei}$  in regression specification (4.1)/(4.2).

To show this, we focus on normal periods, and employ firm-level data from Compustat and CRSP.<sup>22</sup> For each firm  $f$  in industry  $i$ , we define the change in cash in period  $t$  as the increase in the firm's cash holdings from period  $t$  to period  $t + 1$ . We associate the firm's price change frequency with the resided industry's price change frequency. Then, we run a regression according to specification (4.3):

$$\Delta CASH_{i,t}^f = \alpha_0 + \alpha_1 EI_{i,t}^f + \alpha_2 \log(FREQ_i) + \alpha_3 EI_{i,t}^f \times \log(FREQ_i) + \alpha_4 X_{i,t}^f + \varepsilon_{i,t}^f. \quad (4.3)$$

We regress the change in cash ( $\Delta CASH_{i,t}^f$ ) on the amount of equity issuance ( $EI_{i,t}^f$ ), the firm's log price change frequency ( $\log(FREQ_i)$ ), their interaction term ( $EI_{i,t}^f \times \log(FREQ_i)$ ), and a set of control variables ( $X_{i,t}^f$ ) including the firm's average q ( $q_{i,t}^f$ ) and the industry stock return ( $R_{i,t}$ ).

The coefficient on the interaction term,  $\alpha_3$  is to our interest. Table 2 shows that it is negative regardless whether the average q and/or time fixed effects are controlled or not. This indicates that firms with stickier prices are in general using disproportionately more externally financed funds to build up cash reserves.

## 5 Conclusion and General Equilibrium Discussion

We propose a tractable model which demonstrates price setting decisions when a firm is operated in an environment featuring both customer markets and financial frictions, and the impact of price stickiness

<sup>22</sup>Over our sample period 1998-2012, the U.S. economy was in a financial crisis or recession for 21 quarters. The first period was from the third quarter of 1998 to the end of 2001, which was started with the Russian financial crisis, continued with the Argentine economic crisis, and ended up with a recession due to the collapse of the speculative dot-com bubble (Reinhart and Rogoff, 2011, 2014). The second was between the fourth quarter of 2007 and the second quarter of 2009, known as the Great Recession.

Table 2: Firms with sticky prices build up cash reserves through equity financing

	(1)	(2)	(3)	(4)
Amount of issued equity	0.252*** [0.007]	0.251*** [0.007]	0.251*** [0.007]	0.251*** [0.007]
log(price change frequency)	5.536*** [1.260]	5.278** [1.271]	5.659*** [1.262]	5.370** [1.275]
Amount of issued equity × log(price change frequency)	-0.096*** [0.004]	-0.096*** [0.004]	-0.096*** [0.004]	-0.096*** [0.004]
Tobin's q			-26.015 [18.553]	-24.956 [18.881]
Industry stock return			-11.571** [5.676]	-8.098 [7.987]
Time fixed effects	No	Yes	No	Yes
Adj. $R^2$	0.564	0.565	0.566	0.567
Observations	1225	1225	1225	1225

\*\*\* $p < 0.01$ , \*\* $p < 0.05$ , \* $p < 0.1$ . The reported adjusted  $R^2$  for the regressions with time fixed effects is the overall  $R^2$ . Note that the positive and significant coefficient on the log amount of issued equity implies that the increase in cash is larger when more equity is issued. The positive and significant coefficient on the log price change frequency indicates that firms facing stickier prices are building up cash slowly. This is also consistent with our theory as stickier firms have less power to manipulate prices and increase cash revenue. Most importantly, the negative and significant coefficient on the interaction term coincides with the model's predictions, namely, for the same amount of equity financing, firms with stickier prices are building cash reserves more in normal periods.

on a firm's investment and financing.

Our model offers several testable predictions on the interaction between price stickiness and financial frictions, and how they jointly affect a firm's financial and investment decisions. In particular, our model predicts that financially constrained firms have a tendency to set a relatively higher markup, and such tendency is larger when prices are less sticky or external financing costs are high. Moreover, our model predicts that firms facing larger price stickiness are more precautionary in their financial decisions—they tend to delay the payment of dividends and issue less equity. Existing literature (e.g. [Chevalier and Scharfstein, 1996](#); [Gilchrist et al., 2016](#)) and our industry-level analysis provide empirical evidence in line with these predictions. Empirical tests based on more detailed firm-level price data set a research agenda of markup dynamics and the impact of price stickiness.

The interaction between price stickiness and financial frictions highlighted in our model can strengthen countercyclical markups. We analyze markup dynamics in [Appendix C.2](#), which is only intended to be illustrative, since our model is a partial equilibrium model with both the industry average price and the interest rate being exogenously given. Yet, we believe its implications on the cyclicity of markups are robust even after taking into account the feedback effect from firms' optimal decisions on the industry average price. During recessions, financially weak firms have a tendency to increase their product prices to boost revenue. In a symmetric Nash equilibrium, the financial distress can reinforce the motivation for "implicit collusion" during recessions (see [Rotemberg and Saloner, 1986](#)), since there is little loss in each firm's customer base when all firms keep high prices or even raise prices all together

at the same time. Even without “implicit collusion”, under a pure Walrasian equilibrium framework, the higher product prices charged by financially weak firms increase the industry average price. This may further induce financially strong firms to increase their product prices since now they are facing a less elastic demand curve due to a higher industry average price.

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## Appendix

### A Benchmark Cases with Constant Markups

We conduct theoretical experiments by setting two benchmark cases in order to illustrate the role of financial frictions in shaping the dynamics of markups desired by the firm.

#### A.1 The Optimal Static Monopolistic Price

Under the common setup adopted by the New Keynesian literature (e.g. Galí, 2008; Dou et al., 2014, for a review), the firm has no external financing costs and there are no customer flows among sellers. By mapping these assumptions onto our model, we have  $\phi = \gamma = 0$  (i.e. no external financing costs),  $\lambda = 0$  (i.e. no cash holding costs), and  $\alpha = 0$  (i.e. no customer flows among sellers). Thus, on the financial side, the Modigliani and Miller (1958) Theorem holds, and on the product side, the price elasticity of demand only shows up in the short run. The intra-temporal profit optimization, as in traditional New Keynesian (DSGE) models, leads to the equilibrium where the firm chooses  $p = p^* \equiv \frac{\eta}{\eta - 1} \bar{c}$  once it gets the chance to reset its price and keeps this price forever. The desired markup is constant over time and purely determined by the intra-temporal elasticity of demand.<sup>23</sup> In other words, we have  $w_0^P = \infty$ . Since there are no external financing costs, the marginal value of cash held by the firm is one. Thus, it is reasonable to guess that the value function of the firm has the following form

$$u(w, p) \equiv u(p) + w. \quad (\text{A.1})$$

By plugging (A.1) into the coupled ODEs, we can get

$$u(p_H) = \theta(\delta + r) - \theta \sqrt{(\delta + r)^2 - 2[\mu_H - (r + \delta)]} / \theta + 1, \quad \text{and} \quad (\text{A.2})$$

$$u(p_L) = \theta(\delta + r + \xi) - \theta \sqrt{(\delta + r + \xi)^2 - 2[\mu_L + \xi u(p_H) - (r + \delta + \xi)]} / \theta + 1, \quad (\text{A.3})$$

where  $\mu_H \equiv (p_H - \bar{c})\mu(p_H)$  and  $\mu_L \equiv (p_L - \bar{c})\mu(p_L)$  are expected current profits for the firm with  $p_H$  and  $p_L$ , respectively. For illustrative purposes, we assume that  $p_H$  is the optimal static monopolistic product price,  $p_H = p^*$ . It implies that  $\mu_H > \mu_L$ . Therefore, the optimal investment is

$$i(w, p_H) = (r + \delta) - \sqrt{(r + \delta)^2 - 2[\mu_H - (r + \delta)]} / \theta, \quad \text{and}$$

$$i(w, p_L) = (r + \delta + \xi) - \sqrt{(r + \delta + \xi)^2 - 2[\mu_L + \xi u(p_H) - (r + \delta + \xi)]} / \theta.$$

In this case, the steady-state price is deterministic with  $p \equiv p_H$ . We highlight the following implications arising from this simple benchmark case. First, it is apparent that the firm's investment decisions only

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<sup>23</sup>Note that in traditional New Keynesian DSGE models (e.g. Galí, 2008), although the desired markup is constant over time, the realized product price can fluctuate over time which is purely driven by the time-variation in marginal costs. In order to highlight the forces driving time-varying desired markups, we postulate a fixed marginal cost. This helps us to clearly illustrate the key mechanisms of the model.

focus on keeping investment on the optimal growth path, while its price setting decisions focus on maximizing expected current profits. This is called the “growth-profit separation effect”. Second, the desired markup is constant at  $\frac{\eta}{1-\eta}$ . Third, price stickiness has no impact on the firm’s value or its decisions since the optimal price  $p_H$  is constant overtime. Fourth, the efficiency cost of price stickiness is just a pass-through from whatever cost (e.g. menu costs) resulting a sticky product price, if the initial price is not  $p_H$ . That is, the value function is deteriorated exactly by the same amount of menu costs.

At last, in order to make the above equilibrium solution rigorous, we show, in Proposition 1, that it is indeed the case that  $u(p_H) > u(p_L)$ .

**Proposition 1.** *Suppose the parameters satisfy  $(\delta + r)^2 - 2[\mu_H - (r + \delta)] / \theta > 0$ , then  $u(p_L)$  in (A.3) is well defined and  $u(p_H) > u(p_L)$ .*

*Proof.* First, we show that

$$(r + \delta + \xi)^2 - 2[\mu_L + \xi u(p_H) - (r + \delta + \xi)] / \theta > \left\{ \sqrt{(r + \delta)^2 - 2[\mu_H - (r + \delta)] / \theta} + \xi \right\}^2 > 0.$$

Rearranging the terms, we get

$$(r + \delta + \xi)^2 - 2[\mu_L + \xi u(p_H) - (r + \delta + \xi)] / \theta - \left\{ \sqrt{(r + \delta)^2 - 2[\mu_H - (r + \delta)] / \theta} + \xi \right\}^2 = 2(\mu_H - \mu_L) > 0.$$

It is straightforward to see that  $u(p_L)$  converges to  $u(p_H)$  as  $\xi$  goes to infinity. The partial derivative of  $u(p_L)$  with respect to  $\xi$  is

$$\frac{\partial u(p_L)}{\partial \xi} = 1 - \frac{\sqrt{(r + \delta)^2 - 2[\mu_H - (r + \delta)] / \theta} + \xi}{\sqrt{(r + \delta + \xi)^2 - 2[\mu_L + \xi u(p_H) - (r + \delta + \xi)] / \theta}} > 0.$$

Therefore, the value function  $u(p_L)$  is monotonically increasing in  $\xi$ , and  $u(p_H) > u(p_L)$ .  $\square$

## A.2 The Optimal Inter-temporal Monopolistic Price in Customer Market Models

We incorporate the customer market (e.g. Phelps and Winter, 1970; Rotemberg and Woodford, 1993) into traditional New Keynesian models. The only difference from the previous irrelevance benchmark is that now the firm’s customer base affects its profits and customer flows are allowed among sellers, i.e.  $\alpha > 0$  and  $h'(p) < 0$ . The Modigliani and Miller (1958) Theorem still holds, but in this case the price elasticity of demand shows up both in the short run and in the long run. As a result, price setting decisions not only affect the firm’s current profits but also influence the long-run growth rate in customer base, thus leading to an equilibrium price lower than the optimal static monopolistic price  $p^* \equiv \frac{\eta}{\eta-1} \bar{c}$ . In equilibrium, the firm sets  $p_L$  once it gets the chance to reset its price, and the firm keeps the product price at  $p_L$  forever.<sup>24</sup> In other words, we have  $W_0^P = 0$ . As in traditional New Keynesian models, the desired markup is constant over time, however, it is lower than  $p^*$  as emphasized by the traditional customer market model (e.g. Phelps and Winter, 1970; Rotemberg and Woodford, 1993). Since there are

<sup>24</sup>We require  $p_L < p^*$  but  $p_L$  should not be very small so that, for example, the condition (A.7) holds.

no external financing costs, the marginal value of cash is one. Thus, it is reasonable to guess that the value function of the firm has the following form:

$$u(w, p) \equiv u(p) + w. \quad (\text{A.4})$$

By plugging (A.1) into the coupled ODEs, we can get

$$u(p_L) = \frac{\theta[\delta(1-\alpha) + r] + (1-\alpha)}{(1-\alpha)^2} - \frac{\theta\sqrt{[\delta(1-\alpha) + r]^2 - 2(1-\alpha)[\mu_L - r - \delta(1-\alpha)]}/\theta}{(1-\alpha)^2},$$

and

$$u(p_H) = \frac{\theta[\delta(1-\alpha) + r + \xi - \alpha h(p_H)] + (1-\alpha)}{(1-\alpha)^2} - \frac{\theta\sqrt{[\delta(1-\alpha) + r + \xi - \alpha h(p_H) + (1-\alpha)/\theta]^2 - (1 + 2\theta\mu_H + 2\theta\xi u(p_L))(1-\alpha)^2/\theta^2}}{(1-\alpha)^2}. \quad (\text{A.5})$$

Therefore, the optimal investment is

$$i(w, p_L) = \frac{\delta(1-\alpha) + r}{1-\alpha} - \frac{\sqrt{[\delta(1-\alpha) + r]^2 - 2(1-\alpha)[\mu_L - r - \delta(1-\alpha)]}/\theta}{1-\alpha}, \text{ and}$$

$$i(w, p_H) = \frac{\delta(1-\alpha) + r + \xi - \alpha h(p_H)}{(1-\alpha)^2} - \frac{\sqrt{[\delta(1-\alpha) + r + \xi - \alpha h(p_H) + (1-\alpha)/\theta]^2 - (1 + 2\theta\mu_H + 2\theta\xi u(p_L))(1-\alpha)^2/\theta^2}}{1-\alpha}. \quad (\text{A.6})$$

If customer flows are fast enough so that the deterioration of customer base due to a high product price is relatively more significant than the gain from current profits, i.e.

$$(1-\alpha)^2(\mu_H - \mu_L) + \alpha h(p_H) [\theta\delta(1-\alpha) + \theta r + (1-\alpha)] \leq 0. \quad (\text{A.7})$$

In this case, the steady-state price is deterministic with  $p \equiv p_L$ . We highlight four important aspects of this simple benchmark case with the customer market. First, the firm's investment decisions only focus on the optimal growth path, but price setting decisions are affected by both short-run profits and long-run growth rates. When the firm weights more on long-run growth, it is optimal to choose the low price  $p_L$  in order to build up customer base at the cost of reducing current profits. Second, the desired markup is constant at  $\frac{p_L}{c} < \frac{p_H}{c} \equiv \frac{\eta}{1-\eta}$ , which is lower than the optimal static monopolistic markup  $\frac{\eta}{1-\eta}$  due to the existence of customer flows. Third, price stickiness has no impact on the firm's value or its decisions. Lastly, the efficiency cost of price stickiness is just a pass-through to the firm's value from whatever cost (e.g. menu costs) resulting a sticky product price, if the initial price is not  $p_L$ .

Similarly, in order to make the above equilibrium solution rigorous, we show, in Proposition 2, that it

is indeed the case that  $u(p_H) > u(p_L)$ .

**Proposition 2.** *Suppose the parameters satisfy the restriction (A.7) and  $[\delta(1 - \alpha) + r]^2 - 2(1 - \alpha) [\mu_L - r - \delta(1 - \alpha)] / \theta > 0$ , then  $u(p_H)$  in (A.5) is well defined and  $u(p_L) > u(p_H)$ .*

*Proof.* First, it is straightforward to see that

$$\begin{aligned} & [\delta(1 - \alpha) + r + \xi - \alpha h(p_H) + (1 - \alpha)/\theta]^2 - [1 + 2\theta\mu_H + 2\theta\xi u(p_L)] (1 - \alpha)^2 / \theta^2 \\ & - \left\{ \xi - \alpha h(p_H) - \sqrt{[\delta(1 - \alpha) + r]^2 - (1 - \alpha) [\mu_L(1 - \alpha) - \delta(1 - \alpha) - r] / \theta} \right\}^2 \\ & \geq -2\alpha h(p_H) [\delta(1 - \alpha) + r + (1 - \alpha)/\theta] + 2 \frac{(1 - \alpha)^2}{\theta} (\mu_L - \mu_H) > 0. \end{aligned} \quad (\text{A.8})$$

Obviously,  $u(p_H)$  converges to  $u(p_L)$  as  $\xi$  goes to infinity. The partial derivative of  $u(p_H)$  with respect to  $\xi$  is  $\frac{\partial u(p_H)}{\partial \xi}$  which has the following expression

$$\frac{\theta}{(1 - \alpha)^2} \left\{ 1 - \frac{\xi - \alpha h(p_H) - \sqrt{[\delta(1 - \alpha) + r]^2 - (1 - \alpha) [\mu_L(1 - \alpha) - \delta(1 - \alpha) - r] / \theta}}{\sqrt{[\delta(1 - \alpha) + r + \xi - \alpha h(p_H) + (1 - \alpha)/\theta]^2 - [1 + 2\theta\mu_H + 2\theta\xi u(p_L)] (1 - \alpha)^2 / \theta^2}} \right\}.$$

Based on the inequality (A.8) above, we know that  $\frac{\partial u(p_H)}{\partial \xi} > 0$ . Therefore, the value function  $u(p_H)$  is monotonically increasing in  $\xi$ , and  $u(p_L) > u(p_H)$ .  $\square$

## B Data

Our empirical analysis is conducted using quarterly series at the industry level over the period 1998 – 2012. We use three major datasets: Labor Productivity and Costs (LPC) database and the industry-level Producer Price Index (PPI) from the Bureau of Labor Statistics (BLS), the U.S. national and industry economic accounts from the Bureau of Economic Analysis (BEA), and the Compustat/CRSP firm-level quarterly series. In order to measure price stickiness, we also incorporate the median frequency of price change for 14 industries from Nakamura and Steinsson (2008, Table VI), which is collected from the PPI micro dataset.

### B.1 Industry Categories

We focus on the manufacturing sector, and consider broadly defined 18 industries according to the first three digits of the NAICS code. Our industry categories are consistent with BEA industry accounts except for the transportation industry. The transportation industry in BEA accounts is divided into two finer industries, motor vehicles and other transportation. Since the first three digits of the NAICS code for the two sub-industries are the same, we merge the two and use a more general category—transportation industry. Table B.1 lists all the industries and their associated three digits NAICS codes. Among the 18 industries, 10 industries produce durable goods, and the rest produce non-durable goods.

Table B.1: Industry Categories

Industry name	NAICS code
<i>Durable goods</i>	
Wood product	321
Nonmetallic mineral products	327
Primary metals	331
Fabricated metal products	332
Machinery	333
Computer and electronic products	334
Electrical equipment, appliances, and components	335
Transportation	336
Furniture and related products	337
Miscellaneous manufacturing	339
<i>Non-durable goods</i>	
Food and beverage and tobacco products	311, 312
Textile mills and textile product mills	313, 314
Apparel and leather and allied products	315, 316
Paper products	322
Printing and related support activities	323
Petroleum and coal products	324
Chemical products	325
Plastics and rubber products	326

## B.2 Construct Industry-Level Corporate Firms' Variables

Following [Korajczyk and Levy \(2003\)](#), we use firm-level quarterly series from Compustat/CRSP. All series are converted to real values in 1998 dollars using the consumer price index (CPI).

All firms are sorted into their associated industries according to the NAICS code.<sup>25</sup> We focus on relatively large and mature firms that had been existing for at least 16 quarters between 1998 and 2012 and with assets not below the 30% quantile in each industry. As in [Korajczyk and Levy \(2003, footnote 3\)](#), we exclude firms' data if any of the following conditions is satisfied.<sup>26</sup> The capital letters wrote in parentheses refer to the variable name in Compustat dataset.

- (i) Firms with negative selling, general and administrative expenses (XSGAQ), or book assets (ATQ) or property plant and equipment (PPEGTQ).
- (ii) Firms with market-to-book ratios (Tobin's average q, constructed following [Davis, Fama and French](#)

<sup>25</sup>The number of firms in our sample for each industry is provided in parentheses—Wood products (32), Nonmetallic mineral products (44), Primary metals (110), Fabricated metal products (97), Machinery (330), Computer and electronic products (1123), Electrical equipment, appliances, and components (114), Transportation (184), Furniture and related products (27), Miscellaneous manufacturing (257), Food and beverage and tobacco products (145), Textile mills and textile product mills (31), Apparel and leather and allied products (112), Paper products (74), Printing and related support activities (34), Petroleum and coal products (69), Chemical products (821), Plastics and rubber products (83).

<sup>26</sup>Note, if firm  $i$  satisfies any condition in period  $t$ , we only remove the data point for firm  $i$  in period  $t$ . Firm  $i$ 's data points in other periods remain in the sample.

(2000), see below) greater than 19.51 or less than 0.31.

- (iii) Firms with average operating income (OIBDPQ) to book assets ratios (ATQ) over the previous four quarters greater than 0.35 and less than -0.12.
- (iv) Firms with average selling expense (XSGAQ) to sales (SALEQ) ratios over the previous four quarters greater than 1.18 and less than 0.019.
- (v) Firms with accounts receivables (RECTQ) to book assets ratios (ATQ) greater than 0.74.
- (vi) Firms with average depreciation (DPQ) to book assets ratios (ATQ) over the previous four quarters greater than 0.042.
- (vii) Firms with average income taxes (TXTQ) to book assets ratios (ATQ) over the previous four quarters greater than 0.046 and less than -0.016.
- (viii) Firms with one year excess stock returns (see below for the variable construction) greater than 338% and less than -112%.

## Financing variables

### Equity Repurchase

We measure the amount of equity repurchases in period  $t$  as the increase in the amount of treasury stock (TSTKQ) from period  $t - 1$  to period  $t$ .<sup>27</sup> Therefore, for firm  $f$  in industry  $i$ , the repurchase ratio variable in period  $t$ ,  $REPR_{i,t}^f$ , is constructed as  $REPR_{i,t}^f = \frac{TSTKQ_{i,t}^f - TSTKQ_{i,t-1}^f}{ATQ_{i,t}^f}$ .

If either  $TSTKQ_{i,t}^f$  or  $TSTKQ_{i,t-1}^f$  is missing, or if  $TSTKQ_{i,t}^f - TSTKQ_{i,t-1}^f < 0$ , we set  $REPR_{i,t}^f = 0$ . Each industry's repurchase ratio is constructed as the average of all firms' repurchase ratios within that industry weighted by sales. For industry  $i$ ,

$$REPR_{i,t} = \frac{\sum_{f \in \text{industry } i} REPR_{i,t}^f \times SALEQ_{i,t}^f}{\sum_{f \in \text{industry } i} SALEQ_{i,t}^f}.$$

### Equity Issuance

We measure the amount of issued equity as the number of common shares issued (CSHIQ) times the stock price at the end of the quarter (PRCCQ). Since the value of preferred stock is rarely reported in Compustat at the quarterly frequency, we do not distinguish common equity from preferred equity. For firm  $f$  in industry  $i$ , the amount of equity issued is constructed as  $EI_{i,t}^f = CSHIQ_{i,t}^f \times PRCCQ_{i,t}^f$ . The amount of equity issued in period  $t$  is  $\Delta EI_{i,t}^f = EI_{i,t}^f - EI_{i,t-1}^f$  and the equity issuance ratio is  $\Delta EIR_{i,t}^f = \frac{\Delta EI_{i,t}^f}{ATQ_{i,t}^f}$ . Each industry's equity issuance ratio is

<sup>27</sup>We construct equity repurchases based on the change in the amount of treasury stock instead of using total shares repurchased (CSHOPQ) for more accurate measurement.

constructed as the average of all firms' equity issuance ratios within that industry weighted by sales. For industry  $i$ ,

$$\Delta EIR_{i,t} = \frac{\sum_{f \in \text{industry } i} \Delta EIR_{i,t}^f \times SALEQ_{i,t}^f}{\sum_{f \in \text{industry } i} SALEQ_{i,t}^f}.$$

### Debt Change

We measure the total amount of debt as the sum of long term debt (DLTTQ) and the amount of debt in current liabilities (DLCQ). For firm  $f$  in industry  $i$ , the total amount of debt in period  $t$ ,  $DEBT_{i,t}^f$ , is constructed as  $DEBT_{i,t}^f = DLTTQ_{i,t}^f + DLCQ_{i,t}^f$ . The change in debt level in period  $t$ , is constructed as the change in the total amount of debt from period  $t - 1$  to period  $t$ ,  $\Delta DEBT_{i,t}^f = DEBT_{i,t}^f - DEBT_{i,t-1}^f$ . The change in debt ratios is,  $\Delta DEBTR_{i,t}^f = \frac{\Delta DEBT_{i,t}^f}{ATQ_{i,t}^f}$ .

Each industry's change in debt ratios is constructed as the average of all firms' change in debt ratios within that industry weighted by sales. For industry  $i$ ,

$$\Delta DEBTR_{i,t} = \frac{\sum_{f \in \text{industry } i} \Delta DEBTR_{i,t}^f \times SALEQ_{i,t}^f}{\sum_{f \in \text{industry } i} SALEQ_{i,t}^f}.$$

### Other variables

#### Tobin's average q

Following Davis, Fama and French (2000), we construct Tobin's average q as the ratio of market equity over book equity. The book equity is constructed as the sum of stockholder's equity (SEQQ) and deferred taxes and investment tax credit (TXDITCQ) minus preferred/preference stock (PSTKQ). The market equity is constructed as the stock price at the end of the quarter (PRCCQ) times the number of common shares outstanding (CSHOQ). For firm  $f$  in industry  $i$  in period  $t$ , the book equity is  $BK_{i,t}^f = SEQQ_{i,t}^f + TXDITCQ_{i,t}^f - PSTKQ_{i,t}^f$ , the market equity is  $MK_{i,t}^f = PRCCQ_{i,t}^f \times CSHOQ_{i,t}^f$ , and the Tobin's q is  $q_{i,t}^f = \frac{MK_{i,t}^f}{BK_{i,t}^f}$ .

#### Investment rate

We measure the amount of investment using capital expenditure. Compustat database provides the amount of year-to-date capital expenditure (CAPXY). For firm  $f$  in industry  $i$  in period  $t$ , we construct investment rate  $IR_{i,t}^f$  as

$$IR_{i,t}^f = \begin{cases} \frac{CAPXY_{i,t}^f}{ATQ_{i,t}^f} & \text{if } t \text{ is the first quarter of the year.} \\ \frac{CAPXY_{i,t}^f - CAPXY_{i,t=1}^f}{ATQ_{i,t}^f} & \text{otherwise.} \end{cases}$$

#### Cash holdings

Cash holdings are measured as cash and short-term investments (CHEQ), which consist of



cash (CHQ) and short-term investments (IVSTQ). It includes, among others, the following items: cash in escrow; government and other marketable securities; letters of credits; time, demand, and certificates of deposit; restricted cash.

For firm  $f$  in industry  $i$  in period  $t$ , the change in cash is constructed as the increase in the firm's cash holdings from period  $t$  to period  $t + 1$ ,  $\Delta CASH_{i,t}^f = CHEQ_{i,t+1}^f - CHEQ_{i,t}^f$ . The cash to book equity ratio is  $CASHR_{i,t}^f = \frac{CHEQ_{i,t+1}^f}{SEQQ_{i,t}^f + TXDITCQ_{i,t}^f - PSTKQ_{i,t}^f}$ .

### Stock return

The stock return is measured to include both dividend returns and capital gains. We measure dividend returns as the dividend per share at the ex-dividend date (DVPSXQ), and capital gains as the difference in the stock price from period  $t - 1$  to period  $t$  divided by the stock price in period  $t - 1$ . For firm  $f$  in industry  $i$  in period  $t$ , the stock return is constructed as,  $R_{i,t}^f = DVPSXQ_{i,t}^f + \frac{PRCCQ_{i,t}^f - PRCCQ_{i,t-1}^f}{PRCCQ_{i,t-1}^f}$ .

Each industry's stock return is constructed as the average of all firms' stock returns within that industry weighted by sales. For industry  $i$ ,

$$R_{i,t} = \frac{\sum_{f \in \text{industry } i} R_{i,t}^f \times SALEQ_{i,t}^f}{\sum_{f \in \text{industry } i} SALEQ_{i,t}^f}.$$

### Abnormal return

We calculate the average equity abnormal return ( $AR_i$ ) to correct for the observed equity price run-up prior to equity issue announcements. The average abnormal return is approximated by the constant term of the Fama and French 3-factor model. Specifically, for each industry  $i$ , we run the following regression

$$R_{i,t} = \alpha_i + \beta_{i,1}R_t^m + \beta_{i,2}SMB_t + \beta_{i,3}HML_t + \epsilon_t,$$

where  $R^m$  is the excess return of the market portfolio,  $SMB$  stands for "Small (market capitalization) Minus Big", and  $HML$  for "High (book-to-market ratio) Minus Low". The constant term  $\alpha_i$  is used to proxy industry  $i$ 's average equity abnormal return.

## B.3 Measure of Price Stickiness

We follow the empirical literature (e.g. [Cecchetti, 1986](#); [Baharad and Eden, 2004](#); [Bils and Klenow, 2004](#); [Nakamura and Steinsson, 2008](#)) and proxy the degree of price stickiness by measuring how frequently firms change their prices. Firms facing larger price stickiness adjust their prices less frequently. In this sense, the average frequency of price change can be regarded as an inverse measure of price stickiness. There are three factors that largely determine the price change frequency. The first factor is the industry's Herfindahl index. [Gaskins \(1971\)](#) and [Eliashberg and Jeuland \(1986\)](#) argue that firms' pricing decisions

are affected by the threat of competitive entries. Therefore, firms with more monopoly power are facing smaller price stickiness in their products. The second factor is the Tobin's  $q$ , which reflects firms' future perspectives, and has predicative power for future investment opportunities. The third factor is the sensitivity of prices to firms' business conditions. We proxy this factor at the industry level using the sensitivity of industry price indexes with respect to industry specific productivity shocks.

We estimate the industry-level price change frequency in two steps.

In the first step, we collect the value of the three relevant factors for all the 18 industries over the period 1998 – 2012. The Herfindahl index is quite stable over time, and thus we use its value from the 2007 survey conducted by the U.S. census. The industry-level Tobin's  $q$  is estimated using firm-level data from Compustat, as described in Appendix B.2. Appendix B.3.1 elaborates the estimation procedures for the sensitivity of industry price indexes with respect to industry specific productivity shocks.

In the second step, we characterize and estimate the relationship between the price change frequency and the three factors using a linear model. In Nakamura and Steinsson (2008, Table VI), the median price change frequencies measured from the Producer Price Index (PPI) micro-data for 14 industries are provided. These industries are broadly consistent with some of the industries we use, and thus form the basis for our estimation (see Appendix B.3.2 below).

### B.3.1 Estimate the Price Sensitivity of Productivity Shocks

To estimate the sensitivity of price indexes with respect to industry-level productivity shocks, we run the following regression for each industry  $i$  and for the whole economy

$$\pi_{i,t} = \alpha_{i,0} + \alpha_{i,1}\mu_{i,t} + \alpha_{i,2}\log(q_{i,t}/q_{i,t-1}) + \epsilon_{i,t}, \quad (\text{B.1})$$

where  $\mu_{i,t}$  is the estimated innovation in productivity in industry  $i$  in period  $t$ ,  $q_{i,t}$  is the industry  $i$ 's average Tobin's  $q$  in period  $t$ . Thus  $\log(q_{i,t}/q_{i,t-1})$  measures the innovation in industry  $i$ 's Tobin's  $q$ , which reflects the change in investment opportunities. The coefficient  $\alpha_{i,1}$  is to our interest, which measures the price sensitivity of productivity shocks (the change in business conditions) in industry  $i$ .

Industry  $i$ 's productivity innovation,  $\mu_{i,t}$  is estimated by filtering out the economy's Solow residual from industry  $i$ 's Solow residual.<sup>28</sup> To be specific, we assume that an industry's aggregate production function is in its general form,  $Y_i(t) = F[K_i(t), L_i(t), A_i(t)]$ . The output growth can be decomposed into growth in technology, capital and labor:

$$\frac{\dot{Y}_i(t)}{Y_i(t)} = \frac{F_{A,i}(t)A_i(t)}{Y_i(t)} \frac{\dot{A}_i(t)}{A_i(t)} + \frac{F_{K,i}(t)K_i(t)}{Y_i(t)} \frac{\dot{K}_i(t)}{K_i(t)} + \frac{F_{L,i}(t)L_i(t)}{Y_i(t)} \frac{\dot{L}_i(t)}{L_i(t)}. \quad (\text{B.2})$$

With competitive factor markets,  $F_{K,i}(t) = R(t)$  and  $F_{L,i}(t) = w(t)$ . Denote factor shares as  $\alpha_{i,K}(t) \equiv \frac{R(t)K_i(t)}{Y_i(t)}$  and  $\alpha_{i,L}(t) \equiv \frac{w(t)L_i(t)}{Y_i(t)}$ . Denote growth rates of output, capital, and labor as  $g_i(t) \equiv \frac{\dot{Y}_i(t)}{Y_i(t)}$ ,  $g_{i,K}(t) \equiv \frac{\dot{K}_i(t)}{K_i(t)}$ ,  $g_{i,L}(t) \equiv \frac{\dot{L}_i(t)}{L_i(t)}$ . The contribution of technology to growth, or the Solow residual is

<sup>28</sup>Gabaix (2011) applies a similar approach to estimate firm-level granular residuals by taking out the mean growth rate of the sample from each firm's growth rate.

$x_i(t) = \frac{F_{A,i}(t)A_i(t)}{Y_i(t)}$ . We obtain

$$x_i(t) = g_i(t) - \alpha_{i,K}(t)g_{i,K}(t) - \alpha_{i,L}(t)g_{i,L}(t). \quad (\text{B.3})$$

In discrete time, the analog of equation (B.3) is

$$x_i^{t \rightarrow t+1} = g_i^{t \rightarrow t+1} - \bar{\alpha}_{i,K}^{t \rightarrow t+1} g_{i,K}^{t \rightarrow t+1} - \bar{\alpha}_{i,L}^{t \rightarrow t+1} g_{i,L}^{t \rightarrow t+1}, \quad (\text{B.4})$$

where  $\bar{\alpha}_{i,K}^{t \rightarrow t+1} \equiv (\alpha_{i,K}(t) + \alpha_{i,K}(t+1))/2$  and  $\bar{\alpha}_{i,L}^{t \rightarrow t+1} \equiv (\alpha_{i,L}(t) + \alpha_{i,L}(t+1))/2$  are average capital and labor shares from  $t$  to  $t+1$ . The productivity innovation  $\mu_t^i$  is estimated as the residual of the following regression

$$x_i(t) = \beta_i x(t) + \mu_{i,t}, \quad (\text{B.5})$$

where  $x(t)$  refers to the economy's Solow residual, which is estimated using a similar approach. Our estimation of Solow residuals is conducted using annual series from BLS and BEA. The labor growth rate,  $g_{i,L}(t)$ , is constructed using the number of hours-worked from the BLS Labor Productivity and Costs (LPC) database. The capital growth rate,  $g_{i,K}(t)$ , is constructed by adding up the net stock of private equipment (Table 3.2E) and the net stock of private structure (Table 3.2S) from BEA's fixed assets accounts. The output growth rate,  $g_i(t)$ , is constructed using value-added by industry from BEA's industry accounts. The labor share,  $\alpha_{i,L}(t)$ , is constructed by dividing total compensation of employees by value-added. Both variables are available from BEA's Gross-Domestic-Product-(GDP)-by-Industry Data.

We construct industry-level price indexes using PPI (Discontinued Industry Data – (NAICS basis)). There are two issues that complicate our exercise. First, PPI provides price indexes at a finer level than the industry categories we use. Second, for each industry, the price index for some sub-groups are not recorded. To deal with these issues, we simply take the average of price indexes for all available sub-groups, relying on the fact that inflation rates across sub-groups within the same industry are highly correlated.

### B.3.2 Estimate the Price Change Frequency

Nakamura and Steinsson (2008, Table VI) present the median frequencies of price change for 15 major industries over the period 1998 – 2005, which is constructed from the PPI micro dataset. Although industries defined in this paper are based on the SIC code, 14 of them are broadly consistent with our industry categories. For example, the wood product industry in our paper is named lumber and wood product industry in Nakamura and Steinsson (2008), and machinery industry is named machinery and equipment there, etc. Denote  $FREQ_i$  as the price change frequency in industry  $i$ . To obtain the price change frequency for the 4 unknown industries, we estimate a linear specification using the data provided by Nakamura and Steinsson (2008). The estimation result is (with standard errors in parentheses)

Table B.2: Estimated Price Change Frequency at the Industry-level

Our sample		Sample of Nakamura and Steinsson (2008)	
Industry name	Freq.	Industry name	Freq.
<i>Durable goods</i>			
Wood product	1.3	Lumber and wood products	1.3
Nonmetallic mineral products	3.6	Nonmetallic mineral products	4.1
Primary metals	3.7	Metals and metal products	3.8
Fabricated metal products	2.0	N/A	
Machinery	3.9	Machinery and equipment	3.7
Computer and electronic products	16.7	N/A	
Electrical equipment, appliances, and components	8.7	N/A	
Transportation	16.6	Transportation equipment	27.3
Furniture and related products	2.3	Furniture and Household Durables	5.1
Miscellaneous manufacturing	9.5	Miscellaneous manufacturing	16.5
<i>Non-durable goods</i>			
Food and beverage and tobacco products	24.7	Processed foods and feeds	26.3
Textile mills and textile product mills	5.3	Textile products and apparel	2.3
Apparel and leather and allied products	6.3	Hides, skins, leather, and related products	3.8
Paper products	9.9	Pulp, paper and allied products	4.4
Printing and related support activities	2.9	N/A	
Petroleum and coal products	33.2	Fuels and related products and power	48.7
Chemical products	9.4	Chemicals and allied products	6.1
Plastics and rubber products	2.6	Rubber and plastic products	3.2

$$\log(FREQ_i) = \underbrace{-0.8033}_{[0.7206]} + \underbrace{-0.2133}_{[0.1333]} \log(|\alpha_{i,1}|) + \underbrace{1.1988}_{[0.3879]} \log(\bar{q}_i) + \underbrace{0.0045}_{[0.0008]} H_i, \quad Adj.R^2 = 0.7119, \quad (B.6)$$

where  $\bar{q}_i$  is the average industry  $i$ 's Tobin's  $q$  over the period 1998 – 2012.  $H_i$  is industry  $i$ 's Herfindahl index. Table B.2 exhibits the estimated price change frequencies for the 18 industries. The estimation result is pretty reliable due to a large adjusted  $R^2$ . Moreover, the coefficient on the log price sensitivity of productivity shocks is significant at the 10% level, while the coefficients on the log Tobin's average  $q$  and the Herfindahl index are all significant at the 1% level.

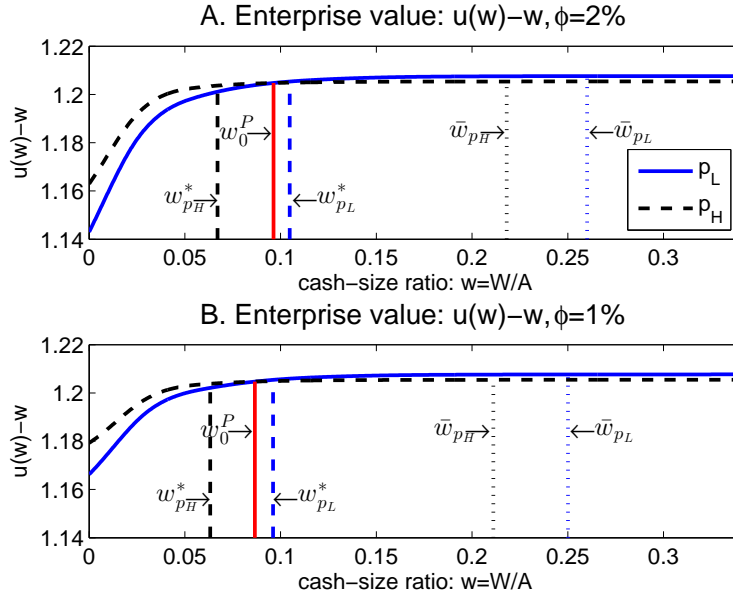


Figure C.1: The firm's enterprise value for different values of the fixed financing cost. Panel A is plotted for  $\phi = 2\%$ , and Panel B is plotted for  $\phi = 1\%$ .

## C Additional Numerical Results

### C.1 The Impact of Financing Costs

In this section, we analyze the impact of the fixed and variable financing costs on the firm's enterprise value, financing, payout, investment, and price setting decisions.

**The Fixed Financing Cost** Figure C.1 presents the firm's normalized enterprise value when the fixed financing cost  $\phi$  varies. When  $\phi$  is increased from 1% to 2%, the enterprise value slightly decreases. Moreover, the payout boundaries and the issuance amounts shift to the right for both  $p_L$  and  $p_H$ , indicating that the firm will hold more cash on its balance sheet. This is because a higher fixed financing cost increases the marginal value of cash by dampening the external financing channel. The price resetting boundary shifts to the right when the fixed financing cost increases. This implies that the firm is more likely to set its price to  $p_H$  when external financing costs are high. The response of investment is similar to Bolton, Chen and Wang (2011), i.e. investment is lower when the fixed financing cost increases (see Figure C.2).

Panel A of Figure C.3 shows that the normalized firm value increases with the fixed financing cost. The underlying force is similar to the one driving the impact of price stickiness. That is, the firm increases its cash holdings (as shown in panel B) due to a higher marginal value of cash when the fixed financing cost increases.<sup>29</sup> Panel C shows that the normalized enterprise value (or the average  $q$ ) decreases as external financing becomes more costly.

<sup>29</sup>Note that the results presented in Panel A of Figure C.3 are about the average firm's value when the firm is in steady state. A larger fixed financing cost indeed reduces the firm's value at the impact of the change in financing costs.

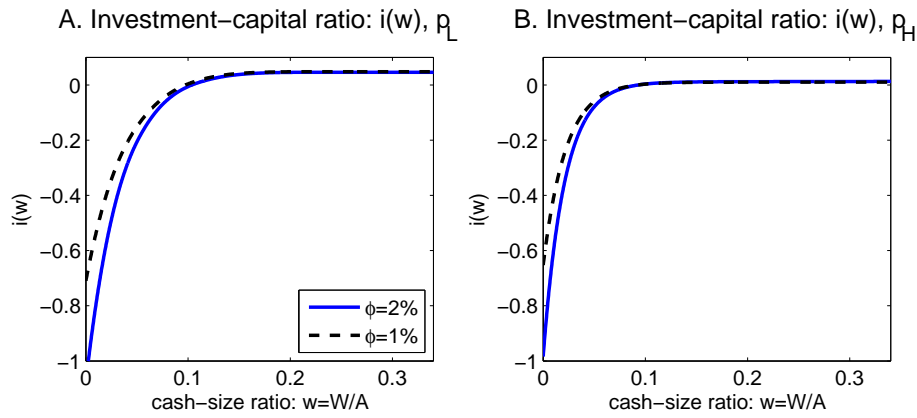


Figure C.2: The firm's investment for different values of the fixed financing cost. Panel A/B plots investment for the  $p_L/p_H$  case.

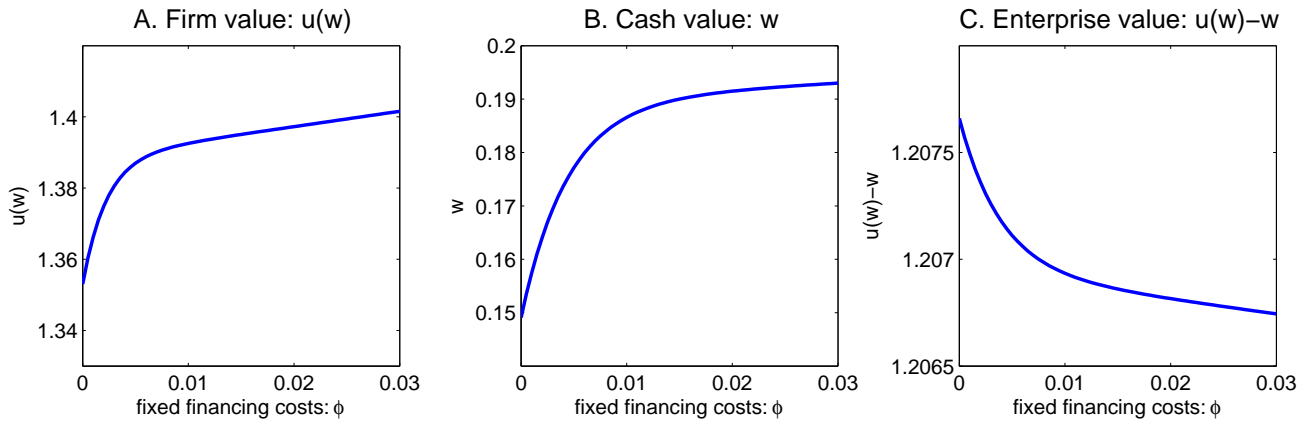


Figure C.3: The average steady-state normalized firm value, cash holdings and normalized enterprise value for different values of the fixed financing cost.

**The Variable Financing Cost** Figure C.4 shows the response of the normalized enterprise value when the variable financing cost varies. Similar to the effect of the fixed financing cost, the normalized enterprise value is lower when the variable financing cost increases.

The firm is less likely to payout when  $\gamma$  increases as the marginal value of cash is higher. This can be seen in Figure C.4, which shows that the payout boundaries (the vertical dotted lines) shift to the right from panel A ( $\gamma = 1\%$ ) to panel C ( $\gamma = 16\%$ ). The optimal issuance amounts are located at the points where the marginal value of cash is equal to one plus the variable financing cost  $\gamma$ . Therefore, when  $\gamma$  increases, the firm issues less equity when hitting the financing boundary (the vertical dashed lines shift to the left).

Notice that the relative locations of the issuance amounts and the price resetting boundary (the vertical solid line) is indeterminate. Panel A plots the case for  $\gamma = 1\%$ . The issuance amount for both the firm with  $p_L$  and the firm with  $p_H$  are to the right of the price resetting boundary ( $w_{p_L}^* > w_{p_H}^* > w_0^P$ ). This suggests that when the firm runs out of cash and issues equity, it takes advantage of the low variable financing cost and raises sufficient external funds in order to not distort its pricing behavior (i.e. set the

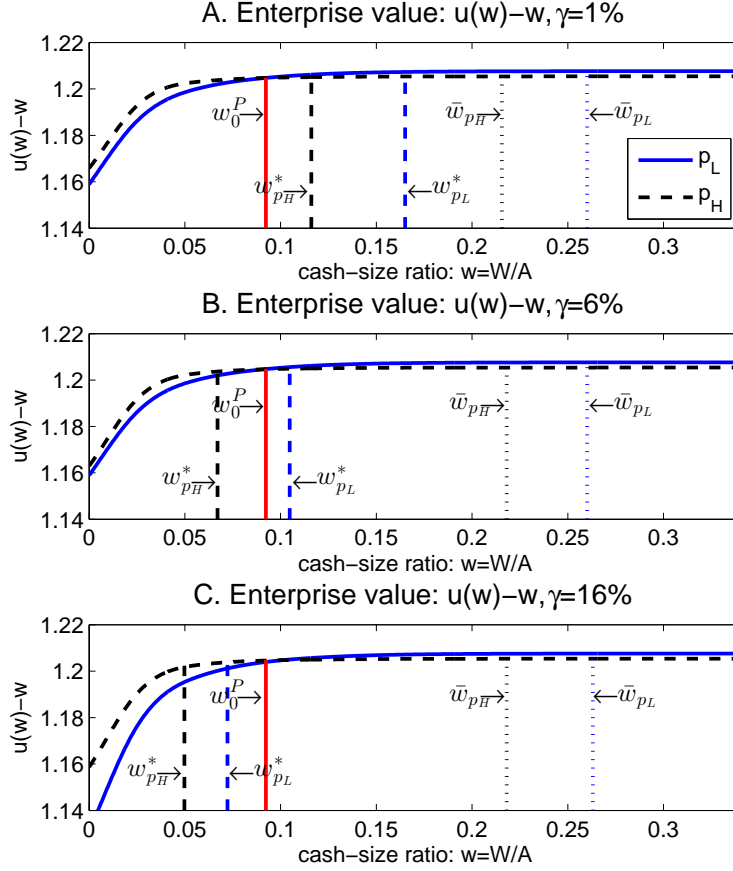


Figure C.4: The firm's enterprise value for different values of the variable financing cost. Panel A is plotted for  $\gamma = 1\%$ , panel B is plotted for  $\gamma = 6\%$ , and panel C is plotted for  $\gamma = 16\%$ .

price to  $p_L$  after external financing). In panel B, the variable financing cost is set at  $\gamma = 6\%$ . In this case, only the firm with  $p_L$  raises sufficient cash through external financing to go beyond the price resetting boundary ( $w_{pH}^* < w_0^P < w_{pL}^*$ ). In panel C, the variable financing cost is high,  $\gamma = 16\%$ . Both the firm with  $p_L$  and  $p_H$  remain in the high price region after external financing ( $w_0^P > w_{pL}^* > w_{pH}^*$ ).

## C.2 Countercyclical Markups

In this section, we present the economy's transitional dynamics to illustrate that our model provides new forces which generate countercyclical markups. Since our model is a partial equilibrium model, without taking into account the feedback effect from firms' optimal decisions on industry average prices or on the equilibrium interest rate, our analysis about countercyclical markups is only intended to be illustrative. A more systematic analysis of the cyclicity of markups based on a general equilibrium model with heterogeneous firms is left for future research.

**Demand Shocks** As elaborated in Section 3, the firm’s desired product price is  $p_H$  when it is financially constrained, since setting a higher price increases the incremental operating revenue. Consider an economy-wide negative demand shock which reduces all firms’ incremental operating revenue. Consequently, firms’ have to run down their cash holdings, and some of them become financially constrained. To prevent costly external financing, these financially constrained firms raise their product prices, resulting in a higher aggregate price level. This generates countercyclical markups/desired markups.

To illustrate this idea, we consider a simple scenario where the steady-state economy, under the benchmark calibration, is hit by an unexpected aggregate negative demand shock which lasts for one quarter.<sup>30</sup> We assume that for firm  $i$ , the nominal shock  $\sigma dZ_t^i$  in equation (2.2) consists of an aggregate component  $\sigma_A dZ_t^A$  and an idiosyncratic component  $\sigma_I dZ_t^i$ , i.e.,  $\sigma dZ_t^i = \sigma_A dZ_t^A + \sigma_I dZ_t^i$  and  $\sigma^2 = \sigma_A^2 + \sigma_I^2$ .  $dZ_t^A$  and  $dZ_t^i$  are standard Brownian motion and are independent from each other. We generate the aggregate demand shock by assuming  $dZ_t^A = -4$  in the first quarter, and  $dZ_t^A = 0$  thereafter. Hence, the first quarter of the economy is hit by a negative aggregate demand shock. Figure C.5 shows the transitional dynamics for the average price and cash holdings of a large number of firms when  $\sigma_I = \sigma_A = \sigma/\sqrt{2}$ . During the first quarter, the negative aggregate demand shock reduces operating revenue and most of the firms have to run down cash to maintain investment. Moreover, firms who find themselves financially constrained increase their prices (i.e. set their prices to  $p_H$ ), which drives up the economy’s price level and generates a higher average markup since the marginal cost of production is unchanged. When the demand shock subsides at the end of the first quarter, financially constrained firms start to accumulate cash and reset their prices to  $p_L$ , allowing the economy’s price level to decrease and gradually converge to its steady-state value.

Notice that the underlying mechanism for the countercyclical markups is different from standard New Keynesian models. In New Keynesian models, a negative demand shock reduces the marginal cost, but as firms cannot adjust prices immediately, the markups would be temporarily high. This relies on the assumptions that the marginal cost of production is increasing in demand and that firms face nominal rigidity. In our model, these two assumptions are not necessary, and we emphasize the roles played by the customer market and financial frictions in generating countercyclical markups. Moreover, customer base can be considered as a force that generates real rigidity. Therefore, the desired markup is also countercyclical in our model, while it is constant in standard New Keynesian models. This addresses the concern raised by Blanchard (2009). Moreover, in contrast to the classical customer market models without financial frictions (e.g. Phelps and Winter, 1970; Phelps, 1992), the countercyclical markups are more robust in our model. As pointed out by Klemperer (1995), the customer market model can generate both countercyclical and procyclical markups depending on the nature of demand shocks. If positive demand shocks come from existing customers increasing their demand, then firms would be motivated to increase their prices to profit from locked-in customers. But, if demand shocks come from the arrival of new customers, then firms would decrease their prices to invest in customer base. In a related paper, Chevalier and Scharfstein (1996) show that the customer market model without financial frictions can

<sup>30</sup>The steady state of the economy is defined as the state where the joint distribution of firms’ cash holdings and product prices is invariant over time. To obtain this, we simulate 500,000 firms for 10 years.



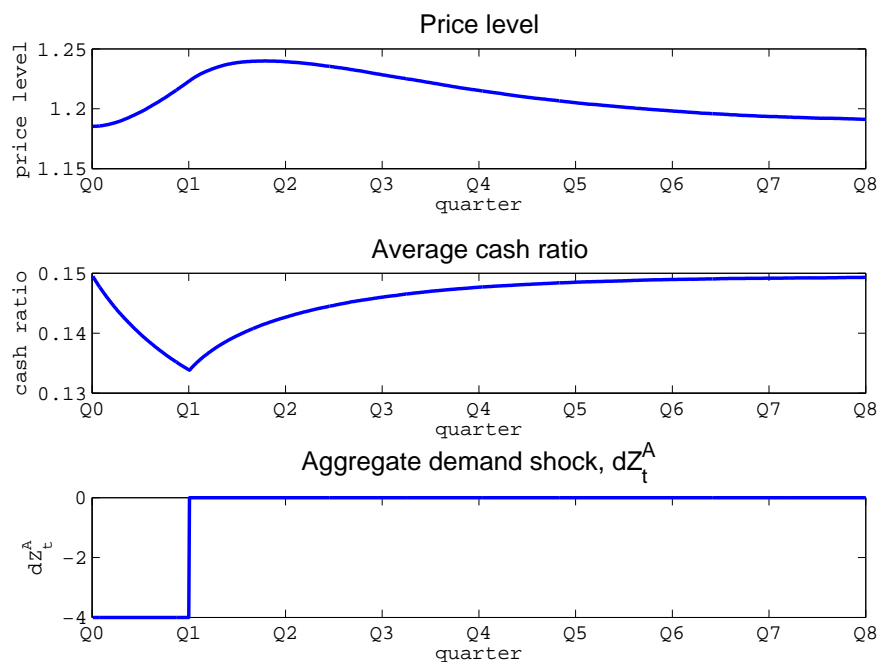


Figure C.5: The dynamics of the average cash ratio and the price level after a demand shock.

only generate procyclical markups.<sup>31</sup> In our model, by introducing financial frictions, the endogenous marginal value of liquidity can interact with the customer market, which generates an additional force pushing towards countercyclical markups.

**Financial Shocks** As shown in Figure C.1, the price resetting boundary shifts to the right when the fixed financing cost increases. This implies that the firm is more likely to set its price to  $p_H$  when external financing costs are high. Since most recessions are associated with a credit crunch in the financial market (Claessens, Kose and Terrones, 2009), the increase in external financing costs provide an additional force pushing towards countercyclical markups.

To show this, we start from the steady-state economy with the benchmark calibration and consider an unexpected one-quarter financial shock which increases the fixed financing cost from 2% to 3%. The results are shown in Figure C.6.

The left three panels of Figure C.6 are based on the benchmark value of parameter  $\zeta = 2.8$ . It shows that firms tend to raise their prices to accumulate more cash when the financial shock arrives in order to avoid costly external financing, resulting in an increase in the average price level. When the financial shock vanishes, both the price level and the average cash ratio converge to their steady-state values. Note that during the first quarter, the price level increases only gradually because firms are facing nominal rigidity. By contrast, when the price is more flexible, the increase in the price level is the highest at the impact of the financial shock, and the price level gradually decreases as firms accumulate more cash and become less financially constrained. This is shown in the right three panels of Figure C.6, where

<sup>31</sup>This is considered as the benchmark model which motivates them to introduce financial frictions in order to generate countercyclical markups. However, in fact, enriching their benchmark model with multiple periods enables countercyclical markups when the increase in demand is from new customers or when demand shocks are persistent.

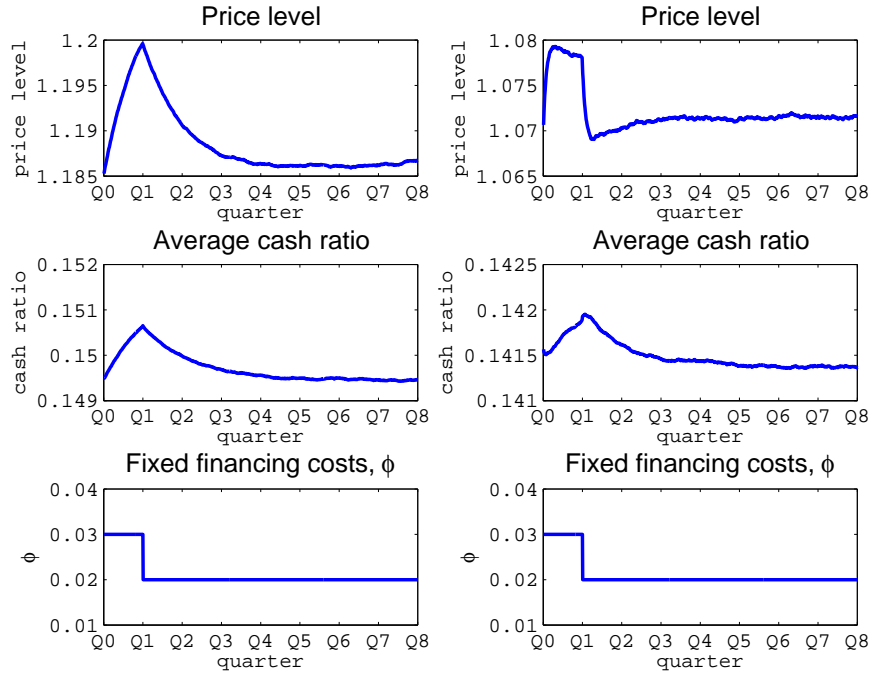


Figure C.6: The dynamics of the average cash ratio and the price level after a financial shock. The left three panels are for the case with  $\zeta = 2.8$ , the right three panels are for the case with  $\zeta = 40$ .

the Calvo parameter is set to  $\zeta = 40$ . Therefore, in our model, nominal rigidity essentially dampens the response of the price level to a financial shock, and the immediate increase in the price level following a negative financial shock is larger when the price is more flexible. Hence, the mechanism delivered by our model has the potential to explain the lack of deflationary pressure during the “Great Recession” in the United States not by appealing to nominal rigidity or large unobservable shocks to the markup.

### C.3 Price Setting and Product Market Characteristics

Following the discussion of equation (2.10), when  $\kappa$  is fixed, parameter  $\nu$  captures how sensitive customer flows are to the change in product prices. By varying the value of  $\nu$ , our model is able to capture markets with different degrees of competition, and further shed light on the firm’s behavior when the characteristics of the underlying product market change. Intuitively, a larger  $\nu$  implies that customer base is more sensitive to the price, reflecting a competitive product market. On the contrary, a smaller  $\nu$  captures the feature of a customer-based product market, which is associated with a higher degree of consumption inertia (or higher switching/information costs). In a market with a relatively small  $\nu$ , the firm loses customer base slowly even if a high price is temporarily charged.

Our model implies that firms in a customer-based product market tend to set their prices to  $p_H$  during a liquidity-constrained period. This is because setting a high price imposes less cost on the firm if the underlying product market is more customer based—the demand is less elastic in the short run and customer base only decreases slowly as consumers are reluctant to change their consumption habits or switch to other brands. However, setting a high price benefits the firm through the current profit

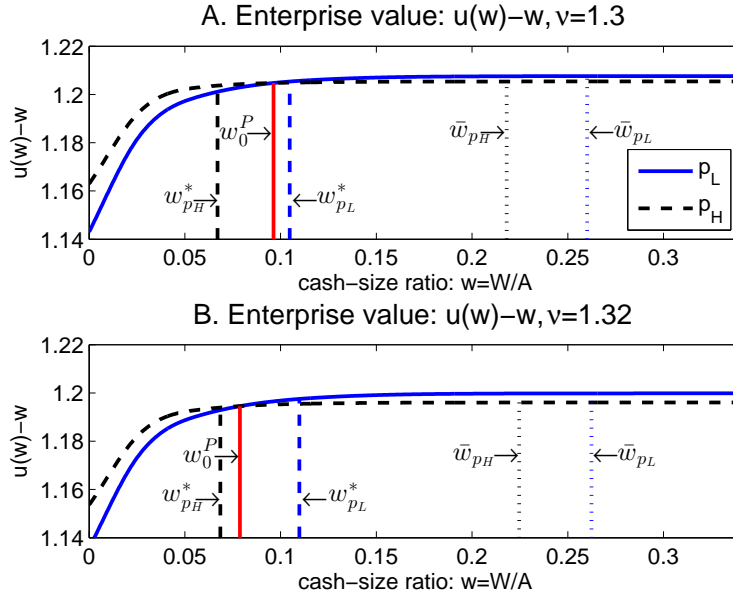


Figure C.7: The firm's enterprise value in different product markets. Panel A is plotted for  $\nu = 1.3$ , and Panel B is plotted for  $\nu = 1.32$ .

channel by increasing short-term operating revenue. Thus, the firm is more inclined to set its price to  $p_H$  when the cash-size ratio is low.

Figure C.7 illustrates this result. As parameter  $\nu$  decreases from 1.32 to 1.3, the price resetting boundary (vertical solid line) shifts to the right, indicating that the firm is more likely to set its price to  $p_H$  when experiencing a liquidity problem. This is consistent with the empirical findings in Gilchrist et al. (2016), which show that firms operating in a more customer-based market (as captured by high SG&A expenses or advertising expenses<sup>32</sup>) raise their product prices relative to the industrial average prices during the recent U.S. financial crisis.

#### C.4 The Volatility of Productivity Shocks

In this section, we discuss the impact of the volatility of productivity shocks on the firm's enterprise value, investment, payout boundaries, optimal issuance amounts, and the price resetting boundary. As shown in Figure C.8 and C.9, when productivity shocks become more volatile (parameter  $\sigma$  increases from 12% to 14%), the firm has a lower enterprise value and lower investment. Moreover, the payout boundaries  $\bar{w}_{p_L}$  and  $\bar{w}_{p_H}$ , and the optimal equity issuance amounts  $w_{p_L}^*$  and  $w_{p_H}^*$  shift to the right, reflecting a higher marginal value of cash. The price resetting boundary ( $w_0^P$ ) shifts to the right, indicating that the firm is more likely to raise its price because a higher volatility of productivity shocks increases the chance of hitting the financing boundary.

<sup>32</sup>Gourio and Rudanko (2014) use SG&A ratios as an indicator for a customer market environment.

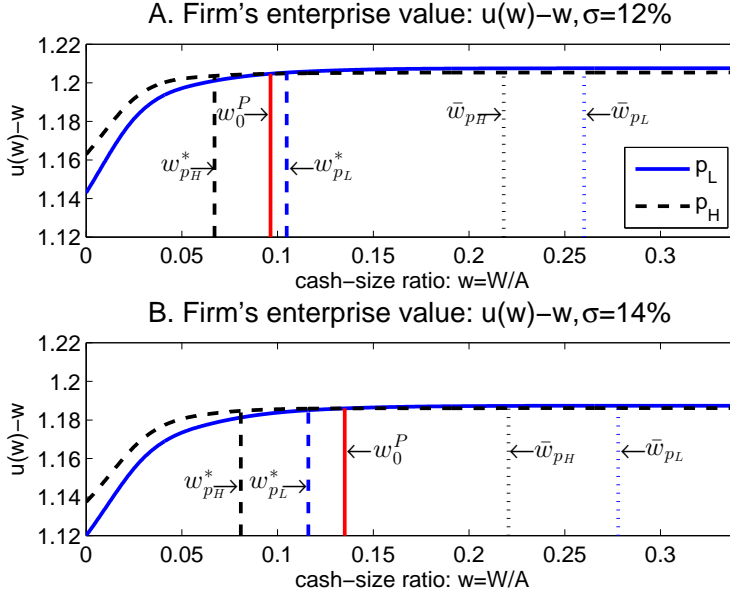


Figure C.8: The firm's enterprise value for different volatilities of productivity shocks. Panel A is plotted for  $\sigma = 12\%$ , and Panel B is plotted for  $\sigma = 14\%$ .

## D A Model with Menu Costs

In our baseline model, price stickiness is modeled as in Calvo (1983). In this section, we modify this aspect by assuming that price adjustment is totally under the control of the firm's manager. However, whenever the price is adjusted, a fixed "menu cost",  $\zeta$  is incurred. This modification enables us to quantitatively measure the direct cost of price stickiness, and to address the concern that the qualitative predictions of the baseline model on the firm's pricing strategy is driven by the mechanical Calvo pricing rule.

Introducing a fixed cost of price adjustment changes the HJB equation (2.14) to the following

$$\begin{aligned}
 rU(A, W, p) = \max_{I, p^+ \in \{p_L, p_H\}} & [\alpha h(p) + (1 - \alpha)(I/K - \delta)] AU_A \\
 & + [(r - \lambda)W + A(p - \bar{c})\mu(p) - \Gamma(I, K, A)] U_W + \frac{1}{2}\sigma^2 A^2 U_{WW} \\
 & + [U(A, W, p^+) - U(A, W, p)] - \zeta \mathbb{1}_{p^+ \neq p},
 \end{aligned} \tag{D.1}$$

where  $\mathbb{1}_{p^+ \neq p}$  is an indicator function, which equals one if  $p^+ \neq p$  and zero otherwise. The financing and payout boundary conditions are the same as the baseline model.

### D.1 Quantitative Results

We set the menu cost parameter  $\zeta = 0.002$  to match the firm's average normalized enterprise value in the menu cost model with the one in the baseline model. The other parameters are set to be the same as the baseline model. Quantitatively, this implies that the menu cost amounts to about 0.15% of the

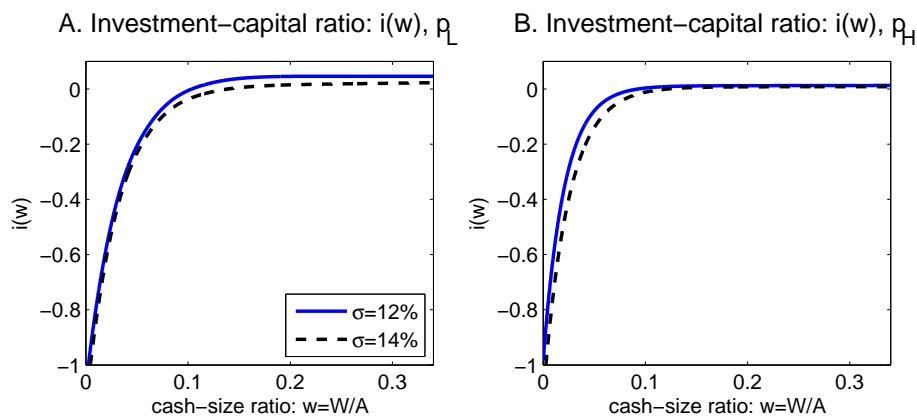


Figure C.9: The firm’s investment for different volatilities of productivity shocks. Panel A/B plots investment for the  $p_L/p_H$  case. Solid/Dashed lines refer to the  $\sigma = 12\%/ \sigma = 14\%$  case.

average normalized enterprise value, or 1% of the firm’s average cash holdings.

Figure D.10 plots the normalized enterprise value for the menu cost model. Similar to the baseline model (Figure 2), the normalized enterprise value of the firm with  $p_L$  is higher than the firm with  $p_H$  when the cash-size ratio is high, and the reverse is true when the cash-size ratio is low. However, the difference in the normalized enterprise value between the firm with  $p_L$  and the firm with  $p_H$  is bounded by the amount of the menu cost. This is intuitive since whenever the difference is larger than the menu cost, the firm will choose to reset its price immediately, which ensures that the resulting difference in the enterprise value cannot exceed the menu cost.

There are two price resetting boundaries in the menu cost model, as marked by the vertical solid lines in the figure. The right boundary ( $w_0^{p_H \rightarrow p_L} = 0.13$ ) captures the threshold of the cash-size ratio at which the firm switches from  $p_H$  to  $p_L$ , while the left boundary ( $w_0^{p_L \rightarrow p_H} = 0.05$ ) captures the threshold where the firm switches from  $p_L$  to  $p_H$ . When the cash-size ratio is between the two boundaries ( $w_0^{p_L \rightarrow p_H} \leq w \leq w_0^{p_H \rightarrow p_L}$ ), the firm is in the “inaction” region and does not change its price at all because the benefit obtained from resetting the price is smaller than the menu cost. When the cash-size ratio is below the left price resetting boundary ( $w < w_0^{p_L \rightarrow p_H}$ ), the firm always sets  $p_H$ , while when the cash-size ratio is above the right price resetting boundary ( $w > w_0^{p_H \rightarrow p_L}$ ), the firm always sets  $p_L$ .

Compared to the baseline model, the menu cost model implies a one-to-one mapping from the cash-size ratio to the product price out of the inaction region. While in the baseline model, the mapping only exists between the cash-size ratio and the “desired” product price. Whether the firm can achieve the desired price or not depends on the arrival of price resetting opportunities, which is out of the manager’s control. Within the inaction region, the firm’s price is indeterminate, depending on the inherited price when the firm first enters the region. The firm facing a larger menu cost (or a stickier price) is associated with a wider inaction region and the shift in issuance amounts and payout boundaries are exactly consistent with the implications of the benchmark model (see Section 3.3) following the same intuitions.

Figure D.11 plots the marginal value of cash (panel A) and the optimal investment-capital ratio (panel B). Consistent with the baseline model, the marginal value of cash is high for the firm with  $p_L$ . However, the difference only shows up within the inaction region. Outside this region, the difference in

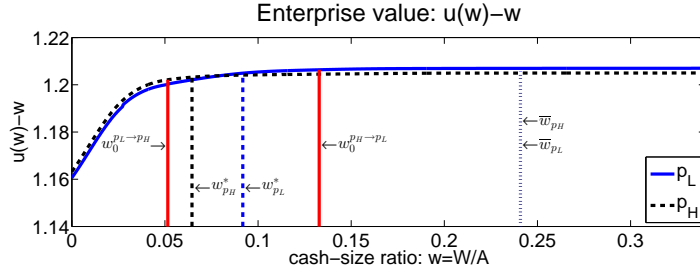


Figure D.10: The enterprise value in the model with menu costs.

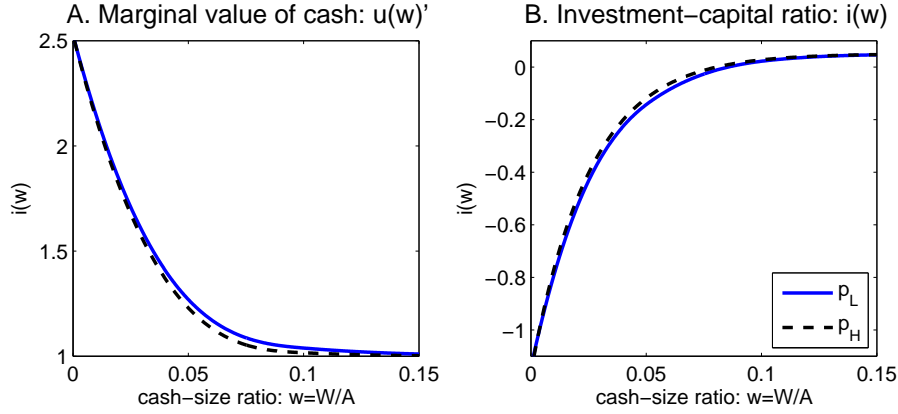


Figure D.11: Marginal value of cash and investment in the model with menu costs.

the enterprise value is locked by the menu cost, resulting in the same marginal value of cash irrespective of the firm's price. Similarly, the firm with  $p_H$  has a higher investment ratio (or a smaller disinvestment ratio) compared to the firm with  $p_L$  only within the inaction region, reflecting the impact of the current profit channel.

## E Numerical Methods

Due to the large non-linearity introduced by price stickiness and financing costs, the solution of the coupled ODE problem is not robust and subject to large numerical errors. To mitigate this problem, we reformulate the continuous time coupled ODEs into a discrete recursive problem, which is solved using a standard dynamic programming method.

Time is discrete with interval  $\Delta$ , The aggregate demand shock  $Q_t$  is i.i.d. and follows a normal distribution,  $N(\mu\Delta, \sigma^2\Delta)$ . In period  $t$ , the state variables for the recursive problem are cash  $W_t$  and effective size  $A_t$ .

Let  $U^L(W, A)$  and  $U^H(W, A)$  be the value functions for the low price and high price, respectively. Price resetting opportunities arrive with probability  $\zeta\Delta$ . We assume that price resetting opportunities arrive before the realization of demand shocks,  $Q$ . The firm makes investment and financing decisions before they know whether they can adjust the price in the current period. After the realization of price resetting opportunities, if the firm obtains the chance to reset its price, either  $p_L$  or  $p_H$  will be chosen to

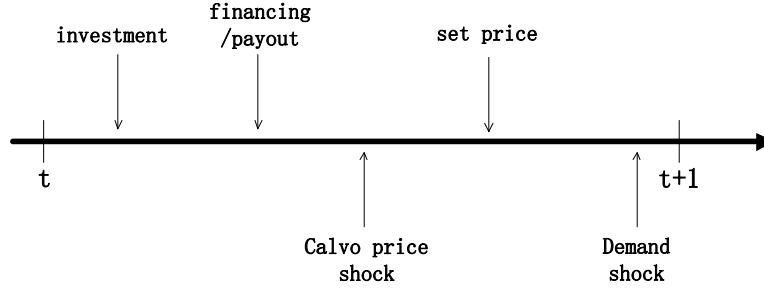


Figure E.12: Timing assumption for the recursive problem.

maximize the objective function. Otherwise, the firm has to stick to the price inherited from the previous period. Demand shocks are realized after price setting decisions are made. Figure E.12 illustrates the timing.

Note that the timing assumption used here is to simplify calculations. Alternatively, we could assume that the firm makes optimal decisions after the realization of demand shocks. However, this would complicate computations because now the firm's optimal decisions also depend on the realized value of demand shocks (see [Dumas and Lyasoff, 2012](#)). When  $\Delta$  approaches zero, the timing assumption does not matter, and the solution will converge to the solution of the coupled ODEs. In our calculation, we set  $\Delta = 0.02$ . The resulted numerical errors are negligible.

The recursive formulation for  $U^L(W, A)$  is

$$U^L(W, A) = \max_{i, U} U + \frac{\xi \Delta}{1 + r \Delta} \max\{E[U^L(W', A')|Q], E[U^H(W'', A'')|Q]\} + \frac{1 - \xi \Delta}{1 + r \Delta} E[U^L(W', A')|Q],$$

subject to

$$\begin{aligned} W' &= W + p_L A Q - g(i) \Delta A + (r - \lambda) \Delta W - U - ((\phi A - \gamma U) 1_{U < 0}), \\ A' &= (1 + h(p_L) \Delta)^\alpha [(1 - \delta \Delta) + i \Delta]^{1 - \alpha} A, \\ W'' &= W + p_H A Q - g(i) \Delta A + (r - \lambda) \Delta W - U - ((\phi A - \gamma U) 1_{U < 0}), \\ A'' &= (1 + h(p_H) \Delta)^\alpha [(1 - \delta \Delta) + i \Delta]^{1 - \alpha} A. \end{aligned}$$

Similarly, for  $V^h(W, A)$ , the recursive formulation is

$$U^H(W, A) = \max_{i, U} U + \frac{\xi \Delta}{1 + r \Delta} \max\{E[U^L(W', A')|Q], E[U^H(W'', A'')|Q]\} + \frac{1 - \xi \Delta}{1 + r \Delta} E[U^H(W'', A'')|Q],$$

subject to

$$\begin{aligned}
W' &= W + p_L A Q - g(i) \Delta A + (r - \lambda) \Delta W - U - ((\phi A - \gamma U) 1_{U < 0}), \\
A' &= (1 + h(p_L) \Delta)^\alpha [(1 - \delta \Delta) + i \Delta]^{1-\alpha} A, \\
W'' &= W + p_H A Q - g(i) \Delta A + (r - \lambda) \Delta W - U - ((\phi A - \gamma U) 1_{U < 0}), \\
A'' &= (1 + h(p_H) \Delta)^\alpha [(1 - \delta \Delta) + i \Delta]^{1-\alpha} A.
\end{aligned}$$

Since  $U(W, A)$  is homogeneous in  $A$ , we write  $U^L(W, A) = u^L(w)A$ ,  $U^H(W, A) = u^H(w)A$ , and define  $w = \frac{W}{A}$ , and  $u = \frac{U}{A}$ .

The recursive problems for the normalized value functions,  $u^L$  and  $u^H$  are

$$\begin{aligned}
u^L(w) &= \max_{i,u} u + \frac{[(1-\delta\Delta)+i\Delta]^{1-\alpha}}{1+r\Delta} \{ (1 - \xi\Delta)(1 + h(p_L)\Delta)^\alpha E[u^L(w')|Q] + \\
&\quad \xi\Delta \max\{ (1 + h(p_L)\Delta)^\alpha E[u^L(w')|Q], (1 + h(p_H)\Delta)^\alpha E[u^H(w'')|Q] \} \},
\end{aligned}$$

subject to

$$\begin{aligned}
w' &= \frac{w + p_L Q - g(i)\Delta + (r - \lambda)\Delta w - u - ((\phi - \gamma u) 1_{u < 0})}{[(1 - \delta \Delta) + i \Delta]^{1-\alpha} (1 + h(p_L) \Delta)^\alpha}, \\
w'' &= \frac{w + p_H Q - g(i)\Delta + (r - \lambda)\Delta w - u - ((\phi - \gamma u) 1_{u < 0})}{[(1 - \delta \Delta) + i \Delta]^{1-\alpha} (1 + h(p_H) \Delta)^\alpha},
\end{aligned}$$

and

$$\begin{aligned}
u^H(w) &= \max_{i,u} u + \frac{[(1-\delta\Delta)+i\Delta]^{1-\alpha}}{1+r\Delta} \{ (1 - \xi\Delta)(1 + h(p_H)\Delta)^\alpha E[u^H(w'')|Q] + \\
&\quad \xi\Delta \max\{ (1 + h(p_L)\Delta)^\alpha E[u^L(w')|Q], (1 + h(p_H)\Delta)^\alpha E[u^H(w'')|Q] \} \},
\end{aligned}$$

subject to

$$\begin{aligned}
w' &= \frac{w + p_L Q - g(i)\Delta + (r - \lambda)\Delta w - u - ((\phi - \gamma u) 1_{u < 0})}{[(1 - \delta \Delta) + i \Delta]^{1-\alpha} (1 + h(p_L) \Delta)^\alpha}, \\
w'' &= \frac{w + p_H Q - g(i)\Delta + (r - \lambda)\Delta w - u - ((\phi - \gamma u) 1_{u < 0})}{[(1 - \delta \Delta) + i \Delta]^{1-\alpha} (1 + h(p_H) \Delta)^\alpha}.
\end{aligned}$$

Table E.3 lists the parameters for the discretization of state space.



Table E.3: Discretization of state space

Parameter	Value	Description
$\Delta$	0.017	Time interval
$n_w$	2000	Number of cash grid points (equally spaced)
$n_i$	1000	Number of investment grid points (equally spaced)
$n_u$	2000	Number of financing grid points (equally spaced)
$[\underline{w} \bar{w}]$	[0 0.50]	Cash range
$[\underline{i} \bar{i}]$	[-1.00 0.50]	Investment range
$[\underline{u} \bar{u}]$	[-0.20 0.20]	Financing range